Chapter 3: Probability

Lesson 3.1: Exploring Probability, page 141

1. e.g., Reverse the rules for Players 1 and 2 on each turn.

2. a) Outcome Table:

		MATT	
		H	T
P	H	HH	HT
A	T	HT	TT
T			

$P(\text{Matt wins}) = \frac{2}{4}$ $P(\text{Pat wins}) = \frac{2}{4}$

$P(\text{Matt wins}) = \frac{1}{2}$ $P(\text{Pat wins}) = \frac{1}{2}$

Each player has an equal chance of winning, so the game is fair.

b) Sample Space:

H	H	H
H	H	T
H	T	H
H	T	T

T	H	H
T	H	T
T	T	H
T	T	T

$P(\text{Treena wins}) = \frac{1}{8}$ $P(\text{Leena wins}) = \frac{1}{8}$ $P(\text{Gina wins}) = \frac{6}{8}$

This game is not fair. Gina has a 6 in 8 chance of winning.

c) Outcome Table:

		Die 1					
		1	**2**	**3**	**4**	**5**	**6**
Die 2	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

$P(\text{Ann wins}) = \frac{15}{36}$ $P(\text{Dan wins}) = \frac{15}{36}$

$P(\text{Ann wins}) = \frac{5}{12}$ $P(\text{Dan wins}) = \frac{5}{12}$

Each player has an equal chance of winning, so the game is fair.

3. No. e.g., A certain chance is 100%. 120% > 100%.

4. Sample Space

Product	1	2	3	4
1	1	2	3	4
2	2	4	6	8
3	3	6	9	12
4	4	8	12	16

Sum	1	2	3	4
1	2	3	4	5
2	3	4	5	6
3	4	5	6	7
4	5	6	7	8

Point Scoring

	1	2	3	4
1	Player 1	Player 1	Player 1	Player 1
2	Player 1	TIE	Player 2	Player 2
3	Player 1	Player 2	Player 2	Player 2
4	Player 1	Player 2	Player 2	Player 2

The probability of Player 1 winning is now $\frac{7}{16}$, or 43.75%. The probability of Player 2 winning is now $\frac{8}{16}$, or 50%. This is not a fair game. Player 2 has more opportunities to win.

Lesson 3.2: Probability and Odds, page 148

1. a) The odds against Marcia passing the driver's test on the first try are 3 : 5.

b) Let A represent Marcia passing her driver's test on the first try.

$P(A) = \frac{5}{5+3}$

$P(A) = \frac{5}{8}$

The probability Marcia will pass her driver's test on the first try is $\frac{5}{8}$, or 0.625.

2. a) The probability the coin is a loonie is $\frac{3}{10}$, or 0.3.

b) The probability the coin is not a loonie is $\frac{7}{10}$, or 0.7.

So, the odds against the coin being a loonie are 7 : 3.

3. a) Let R represent a red card being drawn.

$P(R) = \frac{26}{52}$

$P(R) = \frac{1}{2}$

$P(R) = 0.5$

The probability that Lily draws a red card is 0.5.

b) Let R' represent a card that is not red being drawn.

$P(B) = \frac{(52-26)}{52}$

$P(B) = \frac{26}{52}$

So, the odds in favour of the card being red are 26 : 26, or 1 : 1.

c) Let S represent a spade being drawn.

$P(S) = \frac{13}{52}$ $P(S') = \frac{4-1}{4}$

$P(S) = \frac{1}{4}$ $P(S') = \frac{3}{4}$

The odds against the card being a spade are 3 : 1.

d) Let F represent a face card being drawn.

$$P(F)=\frac{12}{52}$$

$$P(F)=\frac{3}{13}$$

The probability the card drawn is a face card is $\frac{3}{13}$, or about 0.231

4. a) Let A represent apple juice being available at the grocery store.

$$P(A)=\frac{2}{5}$$

$$P(A')=\frac{(5-2)}{5}$$

$$P(A')=\frac{3}{5}$$

The odds in favour of apple juice being available are 2 : 3.

b) The odds against apple juice being available are 3 : 2.

5. Let S represent two students in Mario's class sharing a birthday.

$$P(S)=\frac{7}{7+3}$$

$$P(S)=\frac{7}{10}$$

$$P(S)=0.7$$

The probability two students share a birthday is 0.7.

6. Let C represent Jamia climbing to the top.

a) $$P(C)=\frac{12}{24}$$

$$P(C)=\frac{1}{2}$$

The probability Jamia will make it to the top is $\frac{1}{2}$, or 0.5.

b) $$P(C')=\frac{24-12}{24}$$

$$P(C')=\frac{12}{24}$$

$$P(C')=\frac{1}{2}$$

The odds against Jamia making it to the top are 12 : 12, or 1 : 1.

c) e.g., The odds against and the odds in favour are both 1 : 1.

7. If the probability of snow is 60%, then the probability that of no snow is 40%. Therefore, the odds against snow are 40 : 60, or 2 : 3.

8. If Allan has an 8% chance of having red-green colourblindness, then he has a 92% chance of not having red-green colourblindness. Therefore, the odds in favour of Allan having red-green colourblindness are 8 : 92, or 2 : 23

9. If Katherine had scored 4 times out of 20, then she had not scored 16 times out of 20. Therefore, the odds in favour are 4:16, or 1:4. This can be written as 1 to 4. The probability of her scoring is 4 in 20, or 1 : 5. Therefore, Katherine is not correct.

10. a) If Jason has scored 5 times in his last 10 penalty shots, then he didn't score 5 times. Therefore, the odds in favour of scoring are 5 : 5, or 1 : 1.

b) Gilles blocked 8 of the last 10 penalty shots, and let 2 shots in. Therefore, the odds in favour of Jason scoring on Gilles are 2 : 8, or 1 : 4.

c) e.g., Jason's data reflects his record against all goalies, not just Gilles. Gilles' data suggests that he is better than average at blocking penalty shots.

11. Let S represent a person between the ages of 18 and 35 who uses social networking.

$$P(S)=\frac{31}{(31+19)}$$

$$P(S)=\frac{31}{50}$$

$$P(S)=0.62$$

The probability that a person between the ages of 18 and 35 uses a social networking site is 0.62.

12. Let W represent a win, let L represent a loss, and let T represent a tie.

$$P(W)=\frac{3}{3+2} \qquad P(L)=\frac{1}{1+4} \qquad P(T)=\frac{1}{1+4}$$

$$P(W)=\frac{3}{5} \qquad P(L)=\frac{1}{5} \qquad P(T)=\frac{1}{5}$$

The probability of a win is 3 in 5 (60%), the probability of a loss is 1 in 5 (20%), and the probability of a tie is 1 in 5 (20%). The probabilities add up to 100%.

13. a) 65% of the audience are male, and 35% of the audience are not male. Therefore, the odds in favour of someone watching *Show Trial* being male are 65 : 35, or 13 : 7.

b) 40% of the audience are over 45, and 60% of the audience are under 45, Therefore, the odds in favour of someone watching *Show Trial* being over 45 are 40 : 60, or 2 : 3.

14. a) 70% of the people vaccinated did not get sick, so 30% of the people vaccinated did get sick. So, the odds against getting sick if you are vaccinated are 70 : 30, or 7 : 3.

b) 42% of the people who were not vaccinated got sick, so 58% of the people who were not vaccinated did not get sick. Therefore, the odds against getting sick if you are not vaccinated are 58 : 42, or 29 : 21.

c) The first ratio, 7 : 3, can be written as 49 : 21 by multiplying both numbers by 7. So the odds of getting sick are 49 : 21 and the odds against getting sick are 29 : 21.

d) e.g., Yes, because your chances of not getting sick are much better.

15. a) The odds in favour of getting 7 points, by scoring a touchdown, and the convert, are 5 : 7. The odds in favour of getting 3 points by kicking a field goal, are 5 : 1 or 35 : 7.
b) e.g., The odds in favour of getting 3 points are much better, so the coach should choose the field goal.

16. a) The odds in favour of Eduard winning are 45 : (100 – 45).This is equal to 45 : 55, or 9 : 11. The odds in favour of Julie winning are 35 : (100 – 35). This is equal to 35 : 65, or 7 : 13. The odds in favour of Bill winning are 20 : (100 – 20). This is equal to 20 : 80, or 1 : 4.
b) If Bill's 20% support goes to Julie, then her support will now be 55%, and the odds in favour of Julie winning will be the same as the odds against Edie winning. So, the odds in favour of Julie winning are 11 : 9.

17. a) e.g., The odds in favour of passing are 11 : 9, so the probability of passing is slightly more than 50%. For every 20 people who take the exam, 11 pass.

$$P(\text{pass}) = \frac{11}{20}$$

$$P(\text{pass}) = 0.55$$

$$P(\text{pass}) = 55\%$$

The practice exams cost \$65. The cost to rewrite the exam is \$235.
Yes. If he pays the \$65, he can reduce the 45% chance of having to pay the additional \$235.
b) No. e.g., With odds of 17 : 4, Grant has about a 19% chance of failing even without the practice exams, so he should probably not buy them.
Yes. e.g., With odds of 3 : 7, his chance of failing is 70% without the practice exams, so he should buy them.

18. a) e.g., If the odds for an event are $m : n$, then

$P(A) = \frac{m}{m+n}$ and $P(A') = \frac{n}{m+n}$, so

$P(A') : P(A) = \frac{n}{m+n} : \frac{m}{m+n}$. This ratio is equal to $n : m$.

b) e.g., The probability of the event happening is $\frac{a}{a+b}$.

If the odds in favour of rain tomorrow are 2 : 3, then the probability is $\frac{2}{2+3} = \frac{2}{5}$, or 40%.

c) e.g., The odds against the event happening are $c - a : a$. If the probability of winning the lottery is $\frac{1}{1\,000\,000}$, the odds against are 999 999 : 1.

19. e.g., I prefer using probability because if I express the probability as a percent, it tells me how many times out of a hundred I could expect the event to occur. e.g., I prefer using odds because it compares the chances for and against the event occurring.
20. a) Let A represent a child between the ages of 6 and 18 who will need corrective lenses. Let R represent a girl between the ages of 6 and 18 who will need corrective lenses.

$P(A) = 25.4\%$ $\quad P(R) = \frac{141}{141+100}$

$P(A) = \frac{25.4}{100}$ $\quad P(R) = \frac{141}{241}$

$P(A) = \frac{127}{500}$

$P(A \cap R) = \frac{127}{500} \cdot \frac{141}{241}$

$P(A \cap R) = \frac{17\,907}{120\,500}$

The probability that a randomly selected girl between the ages of 6 and 18 will need corrective lenses is $\frac{17\,907}{120\,500}$, or about 0.149.
b) Let A represent a child between the ages of 6 and 18 who will need corrective lenses. Let B represent a boy between the ages of 6 and 18 who will need corrective lenses.

$P(A) = 25.4\%$ $\quad P(B) = \frac{100}{141+100}$

$P(A) = \frac{25.4}{100}$ $\quad P(B) = \frac{100}{241}$

$P(A) = \frac{127}{500}$

$P(A \cap B) = \left(\frac{127}{500}\right)\left(\frac{100}{241}\right)$

$P(A \cap B) = \frac{12\,700}{120\,500}$

$P(A \cap B) = \frac{127}{1205}$

The probability that a randomly selected 18-year-old boy will need corrective lenses is $\frac{127}{1205}$. So, the odds in favour of a randomly selected 18-year-old boy will need corrective lenses are 127 : (1205 – 127). This is equal to 127 : 1078.

Lesson 3.3: Probabilities Using Counting Methods, page 159

1. Let A represent a passcode with 4 different even digits, and let O represent all 4-digit passcodes.
There are 5 different possibilities for even digits: 0, 2,4,6, and 8. Since these numbers cannot be repeated, the total number of passcodes using 4 different even digits is equal to ${}_5P_4$.

$n(A) = {}_5P_4$

$n(A) = \frac{5!}{(5-4)!}$

$n(A) = \frac{5!}{1!}$

$n(A) = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{1}$

$n(A) = 5 \cdot 4 \cdot 3 \cdot 2$

$n(A) = 120$

The number of 4 digit passcodes is 10^4, because there are 10 possible digits to use in 4 spaces, and repeating is allowed.

$P(A) = \frac{n(A)}{n(O)}$

$P(A) = \frac{120}{10^4}$

$P(A) = \frac{120}{10\ 000}$

The probability that Suri's passcode is made up of four different even digits is $\frac{120}{10\ 000}$, or 0.012 or 1.2%.

2. Let A represent an 8-card hand containing 8 hearts, and let O represent all 8-card hands.
In a standard deck of cards used for Crazy Eights, there are 13 hearts. Therefore, the total number of hands with 8 hearts is ${}_{13}C_8$.

$n(A) = {}_{13}C_8$

$n(A) = \frac{13!}{(13-8)! \cdot 8!}$

$n(A) = \frac{13!}{5! \cdot 8!}$

$n(A) = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 8!}$

$n(A) = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9}{5 \cdot 4 \cdot 3 \cdot 2}$

$n(A) = \left(\frac{12}{4 \cdot 3}\right)\left(\frac{10}{5 \cdot 2}\right)(13)(11)(9)$

$n(A) = 13 \cdot 11 \cdot 9$

The total number of hands containing 8 cards is ${}_{52}C_8$, because there are 52 cards to choose from, and repeating is not possible.

$n(O) = {}_{52}C_8$

$n(O) = \frac{52!}{(52-8)! \cdot 8!}$

$n(O) = \frac{52!}{44! \cdot 8!}$

$n(O) = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44!}{44! \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$n(O) = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}$

$n(O) = \left(\frac{48}{8 \cdot 6}\right)\left(\frac{49}{7}\right)\left(\frac{45}{5}\right)\left(\frac{52}{4}\right)\left(\frac{51}{3}\right)\left(\frac{46}{2}\right)(47)(50)$

$n(O) = 1 \cdot 7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50$

$n(O) = 7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50$

Now determine the probability.

$P(A) = \frac{n(A)}{n(O)}$

$P(A) = \frac{13 \cdot 11 \cdot 9}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50}$

$P(A) = \frac{11}{7 \cdot 17 \cdot 23 \cdot 47 \cdot 50}$

$P(A) = \frac{11}{6\ 431\ 950}$

The probability that a hand will consist of 8 hearts is $\frac{11}{6\,431950}$ or about 0.000 001 71.

3. Let A represent Ben and Jen being chosen as president and secretary, and let O represent all possible committees.
Since order is important, the number of ways in which Ben and Jen can be chosen for president and secretary is ${}_2P_2$.

$n(A) = {}_2P_2$

$n(A) = \frac{2!}{(2-2)!}$

$n(A) = \frac{2!}{0!}$

$n(A) = \frac{2 \cdot 1}{1}$

$n(A) = 2$

The total number of possible committees is ${}_{12}P_2$, because there are 12 people to choose from, and order is important.

$n(O) = {}_{12}P_2$

$n(O) = \frac{12!}{(12-2)!}$

$n(O) = \frac{12!}{10!}$

$n(O) = \frac{12 \cdot 11 \cdot 10!}{10!}$

$n(O) = 12 \cdot 11$

$n(O) = 132$

Now determine the probability.

$$P(A)=\frac{n(A)}{n(O)}$$

$$P(A)=\frac{2}{132}$$

$$P(A)=\frac{1}{66}$$

The probability Ben and Jen will be chosen is $\frac{1}{66}$, or about 0.0152 or 1.52%.

4. a) Let B represent the possibility that only boys will be on the trip, and let O represent all possibilities.

$$n(B)={}_5C_4$$

$$n(B)=\frac{5!}{(5-4)!4!}$$

$$n(B)=\frac{5!}{1!\cdot 4!}$$

$$n(B)=\frac{5\cdot 4!}{1\cdot 4!}$$

$$n(B)=5$$

The number of possibilities of there being all boys on the trip is 5.

$$n(O)={}_{11}C_4$$

$$n(O)=\frac{11!}{(11-4)!4!}$$

$$n(O)=\frac{11!}{7!\cdot 4!}$$

$$n(O)=\frac{11\cdot 10\cdot 9\cdot 8\cdot 7!}{7!\cdot 4!}$$

$$n(O)=\frac{11\cdot 10\cdot 9\cdot 8}{4\cdot 3\cdot 2}$$

$$n(O)=11\cdot 10\cdot 3$$

$$n(O)=330$$

The total number of possibilities for selecting 4 students for the trip is 330.

$$P(B)=\frac{n(B)}{n(O)}$$

$$P(B)=\frac{5}{330}$$

$$P(B)=\frac{1}{66}$$

Therefore, the probability that only boys will be on the trip is $\frac{1}{66}$ or 0.0152 or 1.52%.

b) Let C represent that there is an equal number of boys and girls on the trip.

$$n(C)={}_5C_2\cdot {}_6C_2$$

$$n(C)=\frac{5!}{(5-2)!2!}\cdot\frac{6!}{(6-2)!2!}$$

$$n(C)=\frac{5!}{3!\cdot 2!}\cdot\frac{6!}{4!\cdot 2!}$$

$$n(C)=\frac{5\cdot 4\cdot 3!}{3!\cdot 2\cdot 1}\cdot\frac{6\cdot 5\cdot 4!}{4!\cdot 2\cdot 1}$$

$$n(C)=\frac{5\cdot 4}{2}\cdot\frac{6\cdot 5}{2}$$

$$n(C)=10\cdot 15$$

$$n(C)=150$$

The total number of ways there can be an equal number of boys and girls on the trip is 150.

$$P(C)=\frac{n(C)}{n(O)}$$

$$P(C)=\frac{150}{330}$$

$$P(C)=\frac{5}{11}$$

The probability that an equal number of boys and girls will go on the trip is $\frac{5}{11}$, or about 0.455 or 45.5%.

c) Let D represent that there are more girls than boys on the trip. This is true if 4 girls and no boys are selected and if 3 girls and 1 boy are selected.

$$n(D)={}_6C_4+{}_6C_3\cdot {}_5C_1$$

$$n(D)=\frac{6!}{(6-4)!4!}+\frac{6!}{(6-3)!3!}\cdot\frac{5!}{(5-1)!1!}$$

$$n(D)=\frac{6!}{2!4!}+\frac{6!}{3!3!}\cdot\frac{5!}{4!1!}$$

$$n(D)=\frac{6\cdot 5\cdot 4!}{2!4!}+\frac{6\cdot 5\cdot 4\cdot 3!}{3!3!}\cdot\frac{5\cdot 4!}{4!}$$

$$n(D)=15+20\cdot 5$$

$$n(D)=115$$

There are 115 ways in which there can be more girls than boys on the trip.

$$P(D)=\frac{n(D)}{n(O)}$$

$$P(D)=\frac{115}{330}$$

$$P(D)=\frac{23}{66}$$

The probability that more girls than boys will go is $\frac{23}{66}$ or about 0.348 or 34.8%.

5. Let S represent a password that contains S and Q, and let W represent all possible passwords.
a) The number of possible passwords containing S and Q is ${}_2P_2 \cdot {}_{10}P_3$, or $2! \cdot {}_{10}P_3$. Order in passwords is important, so permutations are used.

$n(S) = 2! \cdot {}_{10}P_3$

$n(S) = 2! \cdot \frac{10!}{(10-3)!}$

$n(S) = 2! \cdot \frac{10!}{7!}$

$n(S) = (2 \cdot 1)\left(\frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}\right)$

$n(S) = 2 \cdot 10 \cdot 9 \cdot 8$

$n(S) = 1440$

The total number of passwords of this form is ${}_{26}P_2 \cdot {}_{10}P_3$.

$n(W) = {}_{26}P_2 \cdot {}_{10}P_3$

$n(W) = \frac{26!}{(26-2)!} \cdot \frac{10!}{(10-3)!}$

$n(W) = \frac{26!}{24!} \cdot \frac{10!}{7!}$

$n(W) = \frac{26 \cdot 25 \cdot 24!}{24!} \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7!}{7!}$

$n(W) = 26 \cdot 25 \cdot 10 \cdot 9 \cdot 8$

$n(W) = 468\,000$

Now determine the probability.

$P(S) = \frac{n(S)}{n(W)}$

$P(S) = \frac{1440}{468000}$

$P(S) = \frac{1}{325}$

The probability that a password chosen at random will include S and Q is $\frac{1}{325}$, or about 0.003 08 or 0.308%.

b) Since repetition is allowed, the number of passwords containing S and Q is ${}_2P_2 \cdot 10^3$, or $2! \cdot 10^3$. The total number of passwords available is $26^2 \cdot 10^3$.

$P(S) = \frac{n(S)}{n(W)}$

$P(S) = \frac{2! \cdot 10^3}{26^2 \cdot 10^3}$

$P(S) = \frac{2!}{26^2}$

$P(S) = \frac{2 \cdot 1}{26^2}$

$P(S) = \frac{2}{676}$

$P(S) = \frac{1}{338}$

The probability that a password chosen at random will include S and Q is $\frac{1}{338}$, or about 0.002 96 or 0.296%.

6. Let F represent the four friends being on the team, and let T represent all the possible teams.
a) The number of ways 4 people can be placed in 4 spots, when order is important, is ${}_4P_4$.

$n(F) = {}_4P_4$

$n(F) = \frac{4!}{(4-4)!}$

$n(F) = \frac{4!}{0!}$

$n(F) = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1}$

$n(F) = 4 \cdot 3 \cdot 2$

$n(F) = 24$

The number of ways 9 people can be placed in 4 spots, when order is important, is ${}_9P_4$.

$n(T) = {}_9P_4$

$n(T) = \frac{9!}{(9-4)!}$

$n(T) = \frac{9!}{5!}$

$n(T) = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5!}$

$n(T) = 9 \cdot 8 \cdot 7 \cdot 6$

$n(T) = 3024$

The probability can now be determined.

$P(F) = \frac{n(F)}{n(T)}$

$P(F) = \frac{24}{3024}$

$P(F) = \frac{1}{126}$

The probability the 4 friends will all be on the team is $\frac{1}{126}$, or about 0.0079 or 0.79%.

b) If only 8 students were applying for the team, then the total number of teams would change from ${}_9P_4$ to ${}_8P_4$.

$n(T) = {}_8P_4$

$n(T) = \frac{8!}{(8-4)!}$

$n(T) = \frac{8!}{4!}$

$n(T) = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!}$

$n(T) = 8 \cdot 7 \cdot 6 \cdot 5$

$n(T) = 1680$

Now determine the probability.

$$P(F)=\frac{n(F)}{n(T)}$$

$$P(F)=\frac{24}{1680}$$

$$P(F)=\frac{1}{70}$$

The probability increases from $\frac{1}{126}$ to $\frac{1}{70}$, or about 0.0143 or 1.43%.

7. Let T represent Tara and Laura being chosen to play in the infield. Let O represent all possible infield lineups. The number of ways to arrange Tara and Laura in the infield positions is $2 \cdot 3!$. The number of ways to arrange the other 7 players in the remaining 2 infield positions is ${}_7P_2$. Therefore, the total number of infield lineups that include Tara and Laura is $(2 \cdot 3!) \cdot {}_7P_2$.

$$n(T)=(2\cdot 3!)\cdot {}_7P_2$$

$$n(T)=(2\cdot 3!)\cdot \frac{7!}{(7-2)!}$$

$$n(T)=(2\cdot 3!)\cdot \frac{7!}{5!}$$

$$n(T)=(2\cdot 3\cdot 2\cdot 1)\cdot \frac{7\cdot 6\cdot 5!}{5!}$$

$$n(T)=2\cdot 3\cdot 2\cdot 7\cdot 6$$

$$n(T)=504$$

The total number of infield lineups possible is ${}_9P_4$.

$$n(O)={}_9P_4$$

$$n(O)=\frac{9!}{(9-4)!}$$

$$n(O)=\frac{9!}{5!}$$

$$n(O)=\frac{9\cdot 8\cdot 7\cdot 6\cdot 5!}{5!}$$

$$n(O)=9\cdot 8\cdot 7\cdot 6$$

$$n(O)=3024$$

Now determine the probability.

$$P(T)=\frac{n(T)}{n(O)}$$

$$P(T)=\frac{504}{3024}$$

$$P(T)=\frac{1}{6}$$

The probability that Tara and Laura will both play in the infield is $\frac{1}{6}$. Therefore the odds in favour of this event are 1 : (6 – 1) or 1 : 5.

8. a) Let Y represent Yuko, Luigi and Justin being chosen, and let T represent all of the ways that treasurer, secretary and liaison can be chosen. In this case, there is only 1 way to achieve the favourable outcome. The number of ways to choose treasurer, secretary and liaison is ${}_{15}P_3$.

$$n(T)={}_{15}P_3$$

$$n(T)=\frac{15!}{(15-3)!}$$

$$n(T)=\frac{15!}{12!}$$

$$n(T)=\frac{15\cdot 14\cdot 13\cdot 12!}{12!}$$

$$n(T)=15\cdot 14\cdot 13$$

$$n(T)=2730$$

Now determine the probability.

$$P(Y)=\frac{n(Y)}{n(T)}$$

$$P(Y)=\frac{1}{2730}$$

The probability is $\frac{1}{2730}$, or about 0.000 366 or 0.0366%.

b) Let Y represent Yuko, Luigi and Justin being chosen, and let L represent all of that ways that three students can be chosen. Again, there is only 1 way to achieve the favourable outcome. The number of ways to choose 3 people from 15 to clean up is ${}_{15}C_3$.

$$n(L)={}_{15}C_3$$

$$n(L)=\frac{15!}{(15-3)!\cdot 3!}$$

$$n(L)=\frac{15!}{12!\cdot 3!}$$

$$n(L)=\frac{15\cdot 14\cdot 13\cdot 12!}{12!\cdot 3\cdot 2\cdot 1}$$

$$n(L)=\frac{15\cdot 14\cdot 13}{3\cdot 2}$$

$$n(L)=\frac{2730}{6}$$

$$n(L)=455$$

Now determine the probability.

$$P(Y)=\frac{n(Y)}{n(L)}$$

$$P(Y)=\frac{1}{455}$$

The probability that Yuko, Luigi and Justin will be picked is $\frac{1}{455}$, or about 0.0022 or 0.220%.

9. a) Let F represent a password that is greater than 5000, and let O represent all possible passwords. There are 4999 4-digit numbers greater than 5000. However, in this range, there are 5 numbers that repeat all 4 digits (5555, 6666, 7777, 8888 and 9999). Therefore, the total number of usable passwords, greater than 5000, is 4994. The total number of four digits possible is 10000. However, there are 10 numbers that repeat all 4 digits. So, there are 9990 usable passwords.

$$P(F)=\frac{n(F)}{n(O)}$$

$$P(F)=\frac{4994}{9990}$$

$$P(F)=\frac{2497}{4995}$$

The probability Lesley's password is greater than 5000 is $\frac{2497}{4995}$, or about 0.4999 or 49.99%.

b) Let B represent a password that begins and ends with the number 4, and let O represent all possible passwords. Because it is assumed that the first and last digits are 4s, only the two middle digits need to be dealt with. Since repeating two digits is allowed, the number of ways to arrange the two middle digits is 10^2, or 100. However, all the digits could be 4, which is not allowed. Therefore, the total number of usable passwords, where the first and last digits are 4s, is 99. Again, the total number of usable passwords is 9990.

$$P(B)=\frac{n(B)}{n(O)}$$

$$P(B)=\frac{99}{9990}$$

$$P(B)=\frac{11}{1110}$$

The probability Lesley's password begins and ends with 4 is $\frac{11}{1110}$, or about 0.0099 or 0.99%.

c) Let E represent a password that begins with an odd number and ends with an even number, and let O represent all possible passwords. If the first digit is odd and the last digit is even, then there is no possible way that all four digits can be the same. The first digit can be one of five odd numbers (1, 3, 5, 7 and 9), and the last digit can be one of 5 even numbers (0, 2, 4, 6 and 8). The two middle digits can be one of ten digits. Therefore the total number of usable passwords that begin with an odd digit and end with an even digit is $5 \cdot 10 \cdot 10 \cdot 5$, or 2500. The total number of usable passwords is 9990.

$$P(E)=\frac{n(E)}{n(O)}$$

$$P(E)=\frac{2500}{9990}$$

$$P(E)=\frac{250}{999}$$

The probability that Lesley's password begins with an odd digit and ends with an even digit is $\frac{250}{999}$, or about 0.2503 or 25.03%.

10. Let T represent three girls and two boys being chosen to form a subcommittee, and let S represent all possible subcommittees.
In this example, order is not important. The number of ways to arrange three girls and two boys from 16 girls and 7 boys is ${}_{16}C_3 \cdot {}_7C_2$.

$$n(T)={}_{16}C_3 \cdot {}_7C_2$$

$$n(T)=\frac{16!}{(16-3)!\cdot 3!}\cdot\frac{7!}{(7-2)!\cdot 2!}$$

$$n(T)=\frac{16!}{13!\cdot 3!}\cdot\frac{7!}{5!\cdot 2!}$$

$$n(T)=\frac{16\cdot 15\cdot 14\cdot 13!}{13!\cdot 3\cdot 2\cdot 1}\cdot\frac{7\cdot 6\cdot 5!}{5!\cdot 2\cdot 1}$$

$$n(T)=\frac{16\cdot 15\cdot 14}{3\cdot 2}\cdot\frac{7\cdot 6}{2}$$

$$n(T)=\frac{3360}{6}\cdot\frac{42}{2}$$

$$n(T)=560\cdot 21$$

$$n(T)=11\,760$$

The number of ways to arrange 23 people in a five-person committee is ${}_{23}C_5$.

$$n(S)={}_{23}C_5$$

$$n(S)=\frac{23!}{(23-5)!\cdot 5!}$$

$$n(S)=\frac{23!}{18!\cdot 5!}$$

$$n(S)=\frac{23\cdot 22\cdot 21\cdot 20\cdot 19\cdot 18!}{18!\cdot 5\cdot 4\cdot 3\cdot 2\cdot 1}$$

$$n(S)=\frac{23\cdot 22\cdot 21\cdot 20\cdot 19}{5\cdot 4\cdot 3\cdot 2}$$

$$n(S)=\frac{4\,037\,880}{120}$$

$$n(S)=33\,649$$

Now determine the probability.

$$P(T)=\frac{n(T)}{n(S)}$$

$$P(T)=\frac{11\,760}{33\,649}$$

$$P(T)=\frac{1680}{4807}$$

The odds in favour that the committee will contain 3 girls and 2 boys is 1680 : 4807 – 1680 or 1680 : 3127.

11. The total number of outcomes is 2^4, or 16. There is only one option where no coins land on tails (all land on head). Therefore, the probability that at least one coin lands tails is $\frac{15}{16}$, or 0.9375 or 93.75%.

12. Let B represent Bilyana and Bojana sitting together, and let S represent all possible seating arrangements.
a) The number of ways to seat Bilyana and Bojana together is $4 \cdot {}_2P_2$, or $4 \cdot 2!$. The number of ways to seat the 3 other people is ${}_3P_3$, or 3!. The number of ways to seat the 5 friends in the row is $4 \cdot 2! \cdot 3!$, or 48.
There are 5! or 120 ways to arrange five people.

$$P(B) = \frac{n(B)}{n(S)}$$

$$P(B) = \frac{48}{120}$$

$$P(B) = \frac{2}{5}$$

The probability that Bilyana and Bojana are sitting together is $\frac{2}{5}$, 0.4 or 40%.

b) The probability that Bilyana and Bojana are not sitting together is the complement of the probability that they are sitting together

$$P(B') = 1 - P(B)$$

$$P(B') = 1 - \frac{2}{5}$$

$$P(B') = \frac{3}{5}$$

The probability Bilyana and Bojana are not sitting together is $\frac{3}{5}$, 0.6 or 60%.

13. a) Let L represent Tanya taking psychology, linear algebra and English in her first term, and let S represent all possible schedules.
There are $4 \cdot 3!$, or 24 ways to arrange psychology, linear algebra and English. The number of ways to fill the fourth slot is 5. Therefore, the number of schedules that contain psychology, linear algebra and English is $24 \cdot 5$, or 120.
The total number of ways to organize 8 courses into 4 slots is ${}_8P_4$.

$$n(S) = {}_8P_4$$

$$n(S) = \frac{8!}{(8-4)!}$$

$$n(S) = \frac{8!}{4!}$$

$$n(S) = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!}$$

$$n(S) = 8 \cdot 7 \cdot 6 \cdot 5$$

$$n(S) = 1680$$

Now determine the probability.

$$P(L) = \frac{n(L)}{n(S)}$$

$$P(L) = \frac{120}{1680}$$

$$P(L) = \frac{1}{14}$$

The probability that three of the four courses will be psychology, linear algebra, and English is $\frac{1}{14}$, or about 0.0714 or 7.14%.
b) Let R represent Tanya taking religion, political studies and biology in her first term, and let S represent all possible schedules.
There are $10 \cdot 3!$, or 60, ways to arrange religion, political studies and biology. The number of ways to fill the remaining two slots is ${}_5P_2$.

$$n(R) = 60 \cdot {}_5P_2$$

$$n(R) = 60 \cdot \frac{5!}{(5-2)!}$$

$$n(R) = 60 \cdot \frac{5!}{3!}$$

$$n(R) = 60 \cdot \frac{5 \cdot 4 \cdot 3!}{3!}$$

$$n(R) = 60 \cdot 5 \cdot 4$$

$$n(R) = 1200$$

The total number of ways to organize 8 courses into 5 slots is ${}_8P_5$.

$$n(S) = {}_8P_5$$

$$n(S) = \frac{8!}{(8-5)!}$$

$$n(S) = \frac{8!}{3!}$$

$$n(S) = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!}$$

$$n(S) = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$$

$$n(S) = 6720$$

Now determine the probability.

$$P(R) = \frac{n(R)}{n(S)}$$

$$P(R) = \frac{1200}{6720}$$

$$P(R) = \frac{5}{28}$$

The probability that three of the five courses will be religion, political studies and biology is $\frac{5}{28}$, or about 0.179 or 17.9%.

14. a) Let A represent a hand that contains the ace to 8 of the same suit, and let O represent all 8-card hands. In this example, order is not important. There are 4 ways to have the ace to 8 of the same suit, because there are 4 suits. There are ${}_{52}C_8$ ways to arrange 52 cards in 8 positions.

$n(O) = {}_{52}C_8$

$$n(O) = \frac{52!}{(52-8)! \cdot 8!}$$

$$n(O) = \frac{52!}{44! \cdot 8!}$$

$$n(O) = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44!}{44! \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(O) = \frac{48}{8 \cdot 6} \cdot \frac{49}{7} \cdot \frac{45}{5} \cdot \frac{52}{4} \cdot \frac{51}{3} \cdot \frac{46}{2} \cdot 47 \cdot 50$$

$n(O) = 7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50$

Now determine the probability.

$$P(A) = \frac{n(A)}{n(O)}$$

$$P(A) = \frac{4}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50}$$

$$P(A) = \frac{2 \cdot 2}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 2 \cdot 25}$$

$$P(A) = \frac{2}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 25}$$

$$P(A) = \frac{2}{376\,269\,075}$$

The probability that an eight-card hand will contain the A, 2, 3, 4, 5, 6, 7 and 8 of the same suit is $\frac{2}{376269075}$ or about 0.000 000 005 32.

b) Let S represent a hand containing 8 cards of the same colour, and let O represent all 8-card hands. The number of ways to arrange 8 cards from 26 cards of the same colour suit is ${}_{26}C_8$. But, both colours are considered. So, the number of ways to have 8 cards of the same colour is $2 \cdot {}_{26}C_8$.

$n(S) = 2 \cdot {}_{26}C_8$

$$n(S) = 2 \cdot \frac{26!}{(26-8)! \cdot 8!}$$

$$n(S) = 2 \cdot \frac{26!}{18! \cdot 8!}$$

$$n(S) = \frac{2 \cdot 26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19 \cdot 18!}{18! \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(S) = \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3}$$

$$n(S) = \frac{26 \cdot 25 \cdot 24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19}{8 \cdot 7 \cdot 3 \cdot 2 \cdot 5 \cdot 4 \cdot 3}$$

$$n(S) = \frac{24}{8 \cdot 3} \cdot \frac{21}{7 \cdot 3} \cdot \frac{25}{5} \cdot \frac{20}{4} \cdot \frac{26}{2} \cdot 19 \cdot 22 \cdot 23$$

$n(S) = 5 \cdot 5 \cdot 13 \cdot 19 \cdot 22 \cdot 23$

Now determine the probability.

$$P(S) = \frac{n(S)}{n(O)}$$

$$P(S) = \frac{5 \cdot 5 \cdot 13 \cdot 19 \cdot 22 \cdot 23}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50}$$

$$P(S) = \frac{5 \cdot 5 \cdot 19 \cdot 22}{7 \cdot 9 \cdot 17 \cdot 47 \cdot 50}$$

$$P(S) = \frac{5 \cdot 5 \cdot 19 \cdot 2 \cdot 11}{7 \cdot 9 \cdot 17 \cdot 47 \cdot 2 \cdot 5 \cdot 5}$$

$$P(S) = \frac{19 \cdot 11}{7 \cdot 9 \cdot 17 \cdot 47}$$

$$P(S) = \frac{209}{50\,337}$$

The probability that all cards in an 8-card hand are the same colour is $\frac{209}{50337}$, or about 0.00415 or 0.415%.

c) Let F represent a hand containing 4 face cards and 4 other cards, and let O represent all 8-card hands. A standard deck of cards contains 12 face cards and 40 other cards. There are ${}_{12}C_4$ ways to choose 4 face cards, and ${}_{40}C_4$ ways to choose 4 other cards. Therefore, the number of ways to have 4 face cards and 4 other cards in a hand is ${}_{12}C_4 \cdot {}_{40}C_4$.

$n(F) = {}_{12}C_4 \cdot {}_{40}C_4$

$$n(F) = \frac{12!}{(12-4)! \cdot 4!} \cdot \frac{40!}{(40-4)! \cdot 4!}$$

$$n(F) = \frac{12!}{8! \cdot 4!} \cdot \frac{40!}{36! \cdot 4!}$$

$$n(F) = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8!}{8! \cdot 4 \cdot 3 \cdot 2 \cdot 1} \cdot \frac{40 \cdot 39 \cdot 38 \cdot 37 \cdot 36!}{36! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(F) = \frac{12 \cdot 11 \cdot 10 \cdot 9}{4 \cdot 3 \cdot 2} \cdot \frac{40 \cdot 39 \cdot 38 \cdot 37}{4 \cdot 3 \cdot 2}$$

$$n(F) = \left(\frac{12}{4 \cdot 3} \cdot \frac{10}{2} \cdot 9 \cdot 11\right) \cdot \left(\frac{40}{4} \cdot \frac{39}{3} \cdot \frac{38}{2} \cdot 37\right)$$

$n(F) = 5 \cdot 9 \cdot 10 \cdot 11 \cdot 13 \cdot 19 \cdot 37$

Now determine the probability.

$$P(F) = \frac{n(F)}{n(O)}$$

$$P(F) = \frac{5 \cdot 9 \cdot 10 \cdot 11 \cdot 13 \cdot 19 \cdot 37}{7 \cdot 9 \cdot 13 \cdot 17 \cdot 23 \cdot 47 \cdot 50}$$

$$P(F) = \frac{5 \cdot 10 \cdot 11 \cdot 19 \cdot 37}{7 \cdot 17 \cdot 23 \cdot 47 \cdot 5 \cdot 10}$$

$$P(F) = \frac{11 \cdot 19 \cdot 37}{7 \cdot 17 \cdot 23 \cdot 47}$$

$$P(F) = \frac{7733}{128\,639}$$

The probability that an 8-card hand contains 4 face cards and 4 other cards is $\frac{7733}{128639}$, or about 0.0601 or 6.01%.

15. Let A represent a 3-letter word, using the letters in the word "CABINET", that contains two vowels and one consonant. Let O represent all 3-letter words that use the letters in the word "CABINET". The word "CABINET" contains 3 vowels and 4 consonants. Therefore, there are ${}_3C_2 \cdot {}_4C_1$ ways to arrange 2 vowels and 1 consonant from 3 vowels and 4 consonants.

$$n(A) = {}_3C_2 \cdot {}_4C_1$$

$$n(A) = \frac{3!}{(3-2)! \cdot 2!} \cdot \frac{4!}{(4-1)! \cdot 1!}$$

$$n(A) = \frac{3!}{1! \cdot 2!} \cdot \frac{4!}{3! \cdot 1!}$$

$$n(A) = \frac{3 \cdot 2!}{2!} \cdot \frac{4 \cdot 3!}{3!}$$

$$n(A) = 3 \cdot 4$$

$$n(A) = 12$$

The number of possible arrangements of 3 letters from 7 letters is ${}_7C_3$.

$$n(O) = {}_7C_3$$

$$n(O) = \frac{7!}{(7-3)! \cdot 3!}$$

$$n(O) = \frac{7!}{4! \cdot 3!}$$

$$n(O) = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2 \cdot 1}$$

$$n(O) = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2}$$

$$n(O) = \frac{210}{6}$$

$$n(O) = 35$$

Now determine the probability.

$$P(A) = \frac{n(A)}{n(O)}$$

$$P(A) = \frac{12}{35}$$

The probability the tiles will consist of 2 vowels and 1 consonant is $\frac{12}{35}$, or about 0.343 or 34.3%.

16. Let D represent an arrangement of the eight dogs such that the bearded collie and the sheltie are together. Let O represent all arrangements.
The number of ways to place the bearded collie and the sheltie together is $7 \cdot 2!$. The number of ways to place the other 6 dogs is 6!. Therefore, the total number of ways to seat all the dogs, with the collie and sheltie together, is $7 \cdot 2! \cdot 6!$, or $2 \cdot 7!$.
There are 8! ways to arrange eight dogs.

$$P(D) = \frac{n(D)}{n(O)}$$

$$P(D) = \frac{2 \cdot 7!}{8!}$$

$$P(D) = \frac{2 \cdot 7!}{8 \cdot 7!}$$

$$P(D) = \frac{2}{8}$$

$$P(D) = \frac{1}{4}$$

The probability that the bearded collie and the sheltie will be next to each other is $\frac{1}{4}$, 0.25 or 25%.

17. Let A represent Maggie and Tanya in the starting lineup.

$$n(A) = {}_6C_2 \cdot {}_4C_1$$

$$n(A) = \frac{6!}{(6-2)! \cdot 2!} \cdot \frac{4!}{(4-1)! \cdot 1!}$$

$$n(A) = \frac{6!}{4! \cdot 2!} \cdot \frac{4!}{3! \cdot 1!}$$

$$n(A) = \frac{6 \cdot 5 \cdot 4!}{4! \cdot 2} \cdot \frac{4 \cdot 3!}{3!}$$

$$n(A) = \frac{6 \cdot 5}{2} \cdot 4$$

$$n(A) = 60$$

The number of ways in which Maggie and Tanya can be in the starting lineup is 60.

$$n(O) = {}_7C_3 \cdot {}_5C_2$$

$$n(O) = \frac{7!}{(7-3)! \cdot 3!} \cdot \frac{5!}{(5-2)! \cdot 2!}$$

$$n(O) = \frac{7!}{4! \cdot 3!} \cdot \frac{5!}{3! \cdot 2!}$$

$$n(O) = \frac{7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 3 \cdot 2} \cdot \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2}$$

$$n(O) = \frac{7 \cdot 6 \cdot 5}{3 \cdot 2} \cdot \frac{5 \cdot 4}{2}$$

$$n(O) = 350$$

The total number of possible starting lineups is 350.

$$P(A) = \frac{n(A)}{n(O)}$$

$$P(A) = \frac{60}{350}$$

$$P(A) = \frac{6}{35}$$

The probability both Maggie and Tanya will be in the starting lineup is $\frac{6}{35}$, or about 0.171 or 17.1%.

18. e.g., I would use permutations in a problem where the order of the items was important, and use combinations in a problem where order was not important. Permutations: Determine the probability that two items are next to each other in a lineup of seven different items that has been placed in a random order. Combinations: Determine the probability that, given eight books, four of which are about math, if I choose five of the eight books, I choose three math books.

19. Let *A* represent a route that passes the pool, and let *O* represent all routes. The route Tuyet takes consists of 5 'rights' and 4 'downs'. Therefore, the total number of possible routes is the total number of moves (9!), dividing out repetition (5! · 4!)

$$n(O) = \frac{9!}{5! \cdot 4!}$$

$$n(O) = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5!}{5! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(O) = \frac{9 \cdot 8 \cdot 7 \cdot 6}{4 \cdot 3 \cdot 2}$$

$$n(O) = \frac{3024}{24}$$

$$n(O) = 126$$

To pass by the pool, Tuyet must first take 3 'rights' and 2 'downs', then must take 2 'rights' and 2 'downs' in any order. Again, repetition must be divided out.

$$n(A) = \frac{5!}{3! \cdot 2!} \cdot \frac{4!}{2! \cdot 2!}$$

$$n(A) = \frac{5 \cdot 4 \cdot 3!}{3! \cdot 2 \cdot 1} \cdot \frac{4 \cdot 3 \cdot 2!}{2! \cdot 2 \cdot 1}$$

$$n(A) = \frac{5 \cdot 4}{2} \cdot \frac{4 \cdot 3}{2}$$

$$n(A) = 5 \cdot 2 \cdot 2 \cdot 3$$

$$n(A) = 60$$

Now determine the probability.

$$P(A) = \frac{n(A)}{n(O)}$$

$$P(A) = \frac{60}{126}$$

$$P(A) = \frac{10}{21}$$

The probability that Tuyet passes the pool is $\frac{10}{21}$, or 47.6%.

20. I know from chapter 4 that the formula to use is $1 - \frac{{}_{365}P_n}{365^n}$, where n is the number of people. I tried a large number, because I thought that there would need to be a lot of people for the percentage to be as high as 80%. I tried $n = 100$.

$1 - \frac{{}_{365}P_{100}}{365^{100}}$ The calculator gave an error message. This number was too high. I tried $n = 30$.

$1 - \frac{{}_{365}P_{30}}{365^{30}} = 0.706...$. This answer was about 71%, so I thought n would be a little more. I tried n = 35.

$1 - \frac{{}_{365}P_{35}}{365^{35}} = 0.814...$ This is a little over 80%. There would need to be 35 people in a room for the probability that two of them have the same birthday is 80%.

History Connection, page 162

Let us now follow in Pascal's footsteps and analyze correctly the chances of winning in these two games.
Single die: Rolling a single die once leads to precisely one of 6 possible outcomes: Exactly one of the numbers 1, 2, 3, 4, 5, or 6 will be on top. The die is called fair, if each of these outcomes is equally likely. Players of dice games usually assume they are using fair dice, so I will assume this too. If I roll a die 4 times, then the total number of all possible outcomes is 6 · 6 · 6 · 6 = 1296
Out of these there are 5 · 5 · 5 · 5 = 625 outcomes with **no** 6 in them. Thus, if I bet on getting at least one 6 when rolling a die 4 times, there are 625 possibilities of losing, and 1296 – 625 = 671 possibilities of winning. So, my chances of winning are higher than my chances of losing.
Two dice: Let us now turn to the two-dice game. Rolling two dice once leads to one of 36 possible outcomes, namely all possible outcomes of rolling die number 1 combined with all possible outcomes of rolling die number two. Thus, if we roll two dice 24 times, then the total number of possible outcomes is 36 · 36 · ... · 36 (36 multiplied with itself 24 times), which is approximately $2.245... \cdot 10^{37}$. Out of these there are 35 · 35 · ... · 35 (35 multiplied with itself 24 times), which is approximately $1.141... \cdot 10^{37}$ outcomes with **no** double 6. Thus, if I gamble on getting at least one double 6 when rolling two dice 24 times, there are approximately $1.141... \cdot 10^{37}$ possibilities of losing, and $1.103... \cdot 10^{37}$ possibilities of winning. This means that the chances of winning with this game are lower than the chances of losing—as the Chevalier De Mere learned the hard way.

Mid-Chapter Review, page 165

1. a) Outcome table

Dice roll	Product	Sum	Who wins?
(1,1,1)	1	3	Erik
(1,1,2)	2	4	Erik
(1,1,3)	3	5	Erik
(1,1,4)	4	6	Erik
(1,2,1)	2	4	Erik
(1,2,2)	4	5	Erik
(1,2,3)	6	6	Ethan
(1,2,4)	8	7	Ethan
(1,3,1)	3	5	Erik
(1,3,2)	6	6	Ethan
(1,3,3)	9	7	Ethan

(1,3,4)	12	8	Ethan
(1,4,1)	4	6	Erik
(1,4,2)	8	7	Ethan
(1,4,3)	12	8	Ethan
(1,4,4)	16	9	Ethan
(2,1,1)	2	4	Erik
(2,1,2)	4	5	Erik
(2,1,3)	6	6	Ethan
(2,1,4)	8	7	Ethan
(2,2,1)	4	5	Erik
(2,2,2)	8	6	Ethan
(2,2,3)	12	7	Ethan
(2,2,4)	16	8	Ethan
(2,3,1)	6	6	Ethan
(2,3,2)	12	7	Ethan
(2,3,3)	18	8	Ethan
(2,3,4)	24	9	Ethan
(2,4,1)	8	7	Ethan
(2,4,2)	16	8	Ethan
(2,4,3)	24	9	Ethan
(2,4,4)	32	10	Ethan
(3,1,1)	3	5	Erik
(3,1,2)	6	6	Ethan
(3,1,3)	9	7	Ethan
(3,1,4)	12	8	Ethan
(3,2,1)	6	6	Ethan
(3,2,2)	12	7	Ethan
(3,2,3)	18	8	Ethan
(3,2,4)	24	9	Ethan
(3,3,1)	9	7	Ethan
(3,3,2)	18	8	Ethan
(3,3,3)	27	9	Ethan
(3,3,4)	36	10	Ethan
(3,4,1)	12	8	Ethan
(3,4,2)	24	9	Ethan
(3,4,3)	36	10	Ethan
(3,4,4)	48	11	Ethan
(4,1,1)	4	6	Erik
(4,1,2)	8	7	Ethan
(4,1,3)	12	8	Ethan
(4,1,4)	16	9	Ethan
(4,2,1)	8	7	Ethan
(4,2,2)	16	8	Ethan
(4,2,3)	24	9	Ethan
(4,2,4)	32	10	Ethan
(4,3,1)	12	8	Ethan
(4,3,2)	24	9	Ethan
(4,3,3)	36	10	Ethan
(4,3,4)	48	11	Ethan
(4,4,1)	16	9	Ethan
(4,4,2)	32	10	Ethan
(4,4,3)	48	11	Ethan
(4,4,4)	64	12	Ethan

$P(\text{sum is higher}) = \frac{13}{64}$

$P(\text{sum is not higher}) = \frac{51}{64}$

This game is not fair. Ethan has the advantage. There are more products that are greater than the sums.

b) Fair. e.g., Coins have an equal chance of landing on heads and landing on tails. Therefore, since each player has the same requirements to win, but with the opposite coin face, this game is fair.

2. a) Outcome Table:

		Spinner 1				
Spinner 2	SUM	1	2	3	4	5
	1	2	3	4	5	6
	2	3	4	5	6	7
	3	4	5	6	7	8
	4	5	6	7	8	9

Total number of outcomes = 20

b) i) $P(\text{sum of 5}) = \frac{4}{20}$

$P(\text{sum of 5}) = \frac{1}{5}$

The probability of getting a sum of 5 is $\frac{1}{5}$, 0.2 or 20%.

ii) $P(\text{sum of 8 or 9}) = \frac{3}{20}$ The probability of getting a sum of 8 or 9 is $\frac{3}{20}$, 0.15 or 15%.

iii) $P(\text{sum of 4}) = \frac{3}{20}$ The probability of getting a sum of 4 is $\frac{3}{20}$, 0.15 or 15%.

3. a) There are 26 black cards in a standard 52-card deck.

$P(\text{black card}) = \frac{26}{52}$

$P(\text{black card}) = \frac{1}{2}$

The probability of drawing a black card is $\frac{1}{2}$, 0.5 or 50%.

b) Since 26 cards are black, then 26 cards are red. Therefore, the odds in favour of the card being black are 26 : 26, or 1 : 1.

c) There are 13 diamonds in a standard deck, so there are 39 cards that are not diamonds. Therefore, the odds against the card being a diamond are 39 : 13, or 3 : 1.

d) Forty cards in each deck are not face cards.

$P(\text{not face card}) = \frac{40}{52}$

$P(\text{not face card}) = \frac{10}{13}$

The probability the card drawn is not a face card is $\frac{10}{13}$, or about 0.769 or 76.9%.

4. $P(\text{win}) = \frac{5}{23+5}$

$P(\text{win}) = \frac{5}{28}$

The probability he will win is $\frac{5}{28}$, or about 0.179 or 17.9%.

5. a) The probability that two of our prime ministers will have the same birthday is slightly less than 50%, or $\frac{1}{2}$.

Therefore, the odds in favour of two prime ministers having the same birthday are slightly less than 1 : 1.

b) The odds against two prime ministers having the same birthday are slightly more than 1 : 1.

c) Yes, Sir John A. Macdonald and Jean Chrétien both share a birthday on January 11. However, Macdonald's official date of birth recorded in the General Register Office in Edinburgh, Scotland is January 10. But, Macdonald celebrated his birthday on January 11.

6. Let *A* represent Frodo, Sam and Aragorn being chosen for president, treasurer and secretary. Let *O* represent all possible selections. The number of ways that Frodo, Sam and Aragorn can be chosen is ${}_3P_3$ or 3!. The number of ways that the 14 people can be chosen for the 3 positions is ${}_{14}P_3$.

$n(A) = {}_{14}P_3$

$n(A) = \frac{14!}{(14-3)!}$

$n(A) = \frac{14!}{11!}$

$n(A) = \frac{14 \cdot 13 \cdot 12 \cdot 11!}{11!}$

$n(A) = 14 \cdot 13 \cdot 12$

$n(A) = 2184$

Now determine the probability.

$P(A) = \frac{n(A)}{n(O)}$

$P(A) = \frac{3!}{2184}$

$P(A) = \frac{3 \cdot 2 \cdot 1}{2184}$

$P(A) = \frac{6}{2184}$

$P(A) = \frac{1}{364}$

The probability Frodo, Sam, and Aragorn will be chosen is $\frac{1}{364}$, or about 0.002 75 or 0.275%.

7. a) Let *A* represent a hand containing the ace, king, queen and jack of the same suit, and let *O* represent all euchre hands. There are 4 different ways to place the A, K, Q, and J of the same suit in a hand (one for each suit). There are 20 ways of placing the fifth card (one for each remaining card). So, there are 80 ways to have the ace, king, queen, and jack of the same suit in a hand. The total number of hands is ${}_{24}C_5$.

$n(O) = {}_{24}C_5$

$n(O) = \frac{24!}{(24-5)! \cdot 5!}$

$n(O) = \frac{24!}{19! \cdot 5!}$

$n(O) = \frac{24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19!}{19! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$n(O) = \frac{24 \cdot 23 \cdot 22 \cdot 21 \cdot 20}{5 \cdot 4 \cdot 3 \cdot 2}$

$n(O) = \frac{5\ 100\ 480}{120}$

$n(O) = 42\ 504$

Now determine the probability.

$P(A) = \frac{n(A)}{n(O)}$

$P(A) = \frac{80}{42504}$

$P(A) = \frac{10}{5313}$

The probability that a dealt hand will contain an ace, king, queen and jack of the same suit is $\frac{10}{5313}$, or about 0.001 882 or 0.1882%.

b) Let *F* represent a hand containing five cards of the same colour, and let *O* represent all euchre hands. There are 12 red cards and 12 black cards. The number of ways a hand can have either colour is ${}_{12}C_5$. Considering both colours, the number of ways a dealt hand can have five cards of the same colour is $2 \cdot {}_{12}C_5$.

$n(F) = 2 \cdot {}_{12}C_5$

$n(F) = 2 \cdot \frac{12!}{(12-5)! \cdot 5!}$

$n(F) = 2 \cdot \frac{12!}{7! \cdot 5!}$

$n(F) = \frac{2 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$

$n(F) = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3}$

$n(F) = \frac{95\ 040}{60}$

$n(F) = 1584$

The total number of hands is ${}_{24}C_5$, or 42 504.

$$P(F) = \frac{n(F)}{n(O)}$$

$$P(F) = \frac{1584}{42504}$$

$$P(F) = \frac{6}{161}$$

The probability that a dealt hand contains five cards of the same colour is $\frac{6}{161}$, or about 0.037 267 or 3.727%.

c) Let F represent a hand containing a four of a kind, and let O represent all euchre hands.
There are 6 different ways to have four of a kind in euchre (one for each rank of card). There are 20 ways to place the fifth card. Therefore, there are 120 ways to have a four of a kind in a dealt hand.
The total number of hands is ${}_{24}C_5$, or 42 504.

$$P(F) = \frac{n(F)}{n(O)}$$

$$P(F) = \frac{120}{42504}$$

$$P(F) = \frac{5}{1771}$$

The probability that a dealt hand will have four of a kind is $\frac{5}{1771}$, or about 0.002 823 or 0.282%.

8. Let A represent a playlist in which Emanuella's six favourite songs are played together. Let O represent all playlists.
The number of ways to arrange Emanuella's favourite songs so that they are together is ${}_6P_6 \cdot 25$, or $6! \cdot 25$.
The number of ways to arrange the other 24 songs is ${}_{24}P_{24}$, or 24!. The number of ways to arrange 30 songs is ${}_{30}P_{30}$, or 30!. Therefore, the total number of playlists that contain the six favourite songs together is $25 \cdot 6! \cdot 24!$, and the total number of playlists is 30!.

$$P(A) = \frac{n(A)}{n(O)}$$

$$P(A) = \frac{6! \cdot 25 \cdot 24!}{30!}$$

$$P(A) = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 25 \cdot 24!}{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26 \cdot 25 \cdot 24!}$$

$$P(A) = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}{30 \cdot 29 \cdot 28 \cdot 27 \cdot 26}$$

$$P(A) = \frac{6 \cdot 5}{30} \cdot \frac{4}{28} \cdot \frac{3}{27} \cdot \frac{2}{26} \cdot \frac{1}{29}$$

$$P(A) = \frac{1}{7 \cdot 9 \cdot 13 \cdot 29}$$

$$P(A) = \frac{1}{23751}$$

The probability that all 6 of Emanuella's favourite songs will be played together is $\frac{1}{23\,751}$, or about 0.000 042 1 or 0.00421%.

9. Let C represent Stella dropping 2 loonies and one other coin. Let O represent all of the combinations of 3 coins possible.
Stella has 6 loonies and 6 other coins. The number of ways that 2 loonies and 1 other coin could be dropped is ${}_6C_2 \cdot {}_6C_1$.

$$n(C) = {}_6C_2 \cdot {}_6C_1$$

$$n(C) = \frac{6!}{(6-2)! \cdot 2!} \cdot \frac{6!}{(6-1)! \cdot 1!}$$

$$n(C) = \frac{6!}{4! \cdot 2!} \cdot \frac{6!}{5! \cdot 1!}$$

$$n(C) = \frac{6 \cdot 5 \cdot 4!}{4! \cdot 2 \cdot 1} \cdot \frac{6 \cdot 5!}{5! \cdot 1}$$

$$n(C) = \frac{6 \cdot 5}{2} \cdot 6$$

$$n(C) = 3 \cdot 5 \cdot 6$$

$$n(C) = 90$$

The total number of ways that 3 coins can be dropped is ${}_{12}C_3$.

$$n(O) = {}_{12}C_3$$

$$n(O) = \frac{12!}{(12-3)! \cdot 3!}$$

$$n(O) = \frac{12!}{9! \cdot 3!}$$

$$n(O) = \frac{12 \cdot 11 \cdot 10 \cdot 9!}{9! \cdot 3 \cdot 2 \cdot 1}$$

$$n(O) = \frac{12 \cdot 11 \cdot 10}{3 \cdot 2}$$

$$n(O) = \frac{1320}{6}$$

$$n(O) = 220$$

Now determine the probability.

$$P(C) = \frac{n(C)}{n(O)}$$

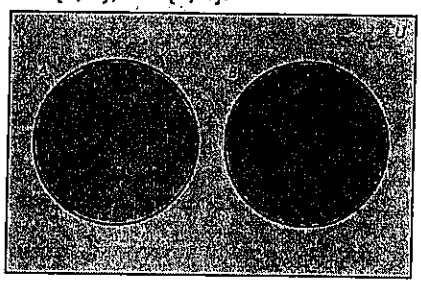
$$P(C) = \frac{90}{220}$$

$$P(C) = \frac{9}{22}$$

The probability that exactly two of the dropped coins are loonies is $\frac{9}{22}$, or about 0.409 or 40.9%.

Lesson 3.4: Mutually Exclusive Events, page 176

1. a) Let A represent rolling a sum of 2. Let B represent rolling a sum of 8.
$A = \{1, 1\}$; $B = \{4, 4\}$.



b) A and B are mutually exclusive, because there is no way for the same two numbers to add up to both 2 and 8.

c) Outcome Table:

		Die 1			
	SUM	1	2	3	4
Die 2	1	2	3	4	5
	2	3	4	5	6
	3	4	5	6	7
	4	5	6	7	8

Total number of outcomes = 16

$$P(\text{sum of 2 or 8}) = \frac{2}{16}$$

$$P(\text{sum of 2 or 8}) = \frac{1}{8}$$

The probability that Zach will roll a sum of 2 or 8 is $\frac{1}{8}$, 0.125 or 12.5%.

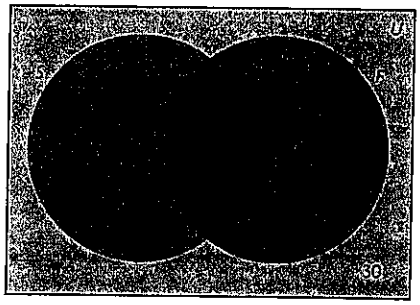
d) Doubles: {(1,1), (2,2), (3,3), (4,4)}
Sum of 6: {(2,4), (3,3), (4,2)}

$$P(\text{doubles or sum of 6}) = \frac{6}{16}$$

$$P(\text{doubles or sum of 6}) = \frac{3}{8}$$

The probability Zach will roll doubles or a sum of 6 is $\frac{3}{8}$, 0.375 or 37.5%.

2. a)



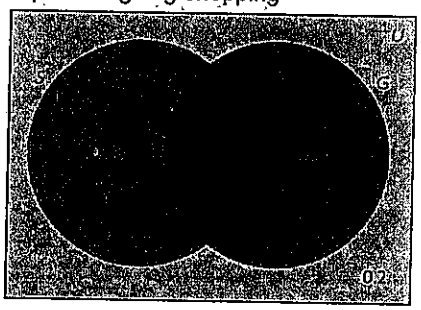
b) The events are not mutually exclusive. For example, the king of spades is both a face card and a spade.

c) $P(\text{face card or spade}) = \frac{22}{52}$

$$P(\text{face card or spade}) = \frac{11}{26}$$

The probability of drawing a face card or spade is $\frac{11}{26}$, or about 0.423 or 42.3%.

3. a) Let G represent going to the gym, and let S represent going shopping



b) No, these events are not mutually exclusive.
$P(G) + P(S) + P(\text{neither}) = 0.75 + 0.4 + 0.2 = 1.35$.
This exceeds 1, so there are favourable outcomes for events G and S that are common.

c) $P(G \cup S) = 1 - P(\text{neither})$
$P(G \cup S) = 1 - 0.2$
$P(G \cup S) = 0.8$
The probability Maria will do at least one of these activities on Saturday is 0.8 or 80%.

4. a) No. e.g., 2 is both an even number and a prime number.
b) Yes. e.g., You cannot roll a sum of 10 and a roll of 7 at the same time.
c) Yes. e.g., You cannot walk and ride to school at the same time.

5. a) $P(A) = \frac{n(A)}{n(C)}$

$$P(A) = \frac{144\ 945}{389\ 045}$$

$$P(A) = \frac{28\ 989}{77\ 809}$$

The probability a person who is Métis lives in Alberta or British Columbia is $\frac{28989}{77809}$, or about 0.373 or 37.3%.

b) $P(M) = \frac{n(M)}{n(C)}$

$P(M) = \frac{119\ 920}{389\ 045}$

$P(M) = \frac{23\ 984}{77\ 809}$

The probability that a person who is Métis lives in Manitoba or Saskatchewan is $\frac{23984}{77809}$, or about 0.308 or 30.8%.

c) Yes, because these two events are mutually exclusive, so $P(A \cap M)$ is equal to 0.

d) The odds in favour of a person who is Métis living in one of the four western provinces are 264 865 : 124 180, or 52 973 : 24 836.

6. There are 12 stuffed dogs and bears, and 28 other prizes. Therefore, the odds in favour of winning either a stuffed dog or a stuffed bear are 12 : 28, or 3 : 7.

7. a) Let F represent the dice rolls with a sum of 5, let N represent the dice rolls with a sum of 9. Let O represent the set of all dice rolls.

F: {(1,4), (2,3), (3,2), (4,1)}

N: {(3,6), (4,5), (5,4), (6,3)}

$n(F \cup N) = 8$

$n(O) = 36$

The odds against the sum equaling 5 or 9 are 28 : 8, or 7 : 2.

b) Let E represent the dice rolls in which both dice are even numbers, and let S represent the dice rolls with a sum of 8. Let O represent the set of all dice rolls.

E: {(2,2), (2,4), (2,6), (4,2), (4,4), (4,6), (6,2), (6,4), (6,6)}

S: {(2,6), (3,5), (4,4), (5,3), (6,2)}

(2,6), (4,4) and (6, 2) are in both sets.

$n(E \cup S) = n(E) + n(S) - n(E \cap S)$

$n(E \cup S) = 9 + 5 - 3$

$n(E \cup S) = 11$

$n(O) = 36$

The odds against both dice being even numbers or the sum being 8 are 25 : 11.

8. a) Let S represent studying and V represent playing video games.

$P(S \cup V) = P(S) + P(V) - P(S \cap V)$

$0.8 = 0.4 + 0.6 - P(S \cap V)$

$P(S \cap V) = 0.2$

The probability that John will do both activities is 0.2 or 20%.

b) No. Since $P(S \cap V) \neq 0$, then $n(S \cap V) = 0$, so the sets of favourable outcomes for S and V are not disjoint.

9. a) No. e.g., One athlete won two ore more medals at the Summer and Winter Olympics.

b) Total number of medal winners = 307

The odds in favour of a Canadian medal winner winning two or more medals at the Summer Olympics are 21 : (307 – 21) or 21 : 286.

c) $n(S \cup W) = 20 + 47 + 1$

$n(S \cup W) = 68$

Total number of medal winners = 307.

The odds in favour of the athlete having won two or more medals is 68 : (307 – 68) or 68 : 239.

10. e.g., Tricia has a probability of 0.3 of cycling to school on any given day, and a probability of 0.2 of getting a ride from her older brother, Steve. Otherwise, she walks to school. What is the probability that she does not walk to school on any given day? (0.5)

11. e.g., There are 67 Grade 10 students that take art and 37 that take photography. If there are 87 students, how many take both? (17)

12. a) Let G represent wearing glasses and H represent having a hearing loss.

If 68% of seniors have a hearing loss, and 10% of these people do not wear glasses, then 10% · 68%, or 6.8% of seniors have a hearing loss but do not wear glasses. This means that 61.2% of seniors wear glasses and have a hearing loss.

$P(H \setminus G) = 6.8\%$

$P(G \cap H) = 61.2\%$

$P(G \setminus H) = P(G) - P(G \cap H)$

$P(G \setminus H) = 76\% - 61.2\%$

$P(G \setminus H) = 14.8\%$

The probability this person will wear glasses and not have hearing aids is 14.8%.

b) Let G represent wears glasses and H represent having a hearing loss.

$P((G \cup H)') = 100\% - (76\% + 6.8\%)$

$P((G \cup H)') = 17.2\%$

The probability that this person will not wear glasses and not have hearing loss is 17.2%.

13. a) Let E represent the eights, and let K represent the kings. Let O represent all cards.

$n(E) = 4$

$n(K) = 4$

$n(E \cap K) = 0$

$n(E \cup K) = n(E) + n(K)$

$n(E \cup K) = 8$

$n(O) = 52$

$P(E \cup K) = \frac{n(E \cup K)}{n(O)}$

$P(E \cup K) = \frac{8}{52}$

$P(E \cup K) = \frac{2}{13}$

The probability of drawing an eight or a king is $\frac{2}{13}$, or about 0.154 or 15.4%.

b) Let R represent the red cards, and let F represent the face cards. Let O represent all cards.

$n(R) = 26$
$n(F) = 12$
$n(R \cap F) = 6$
$n(R \cup F) = n(R) + n(F) - n(R \cap F)$
$n(R \cup F) = 26 + 12 - 6$
$n(R \cup F) = 32$
$n(O) = 52$

$$P(R \cup F) = \frac{n(R \cup F)}{n(O)}$$

$$P(R \cup F) = \frac{32}{52}$$

$$P(R \cup F) = \frac{8}{13}$$

The probability of drawing a red card or a face card is $\frac{8}{13}$, or about 0.615 or 61.5%.

14. Let D represent the households that have one or more dogs, and let C represent the households that have one of more cats. Let O represent all Prairie households. $P(D) = 37\%$; $P(C) = 31\%$

a) $P(D \cup C) = 100\% - P((D \cup C)')$
$P(D \cup C) = 100\% - 47\%$
$P(D \cup C) = 53\%$

The probability a Prairie household has a cat or dog is 53%.

b) $P(D \cup C) = P(D) + P(C) - P(D \cap C)$
$53\% = 37\% + 31\% - P(D \cap C)$
$53\% = 68\% - P(D \cap C)$
$P(D \cap C) = 15\%$
$P(C \setminus D) = P(C) - P(D \cap C)$
$P(C \setminus D) = 31\% - 15\%$
$P(C \setminus D) = 16\%$

The probability that a Prairie household has one or more cats, but no dogs, is 16%.

c) $P(D \setminus C) = P(D) - P(D \cap C)$
$P(D \setminus C) = 37\% - 15\%$
$P(D \setminus C) = 22\%$

The probability that a Prairie household has one or more dogs, but no cats, is 22%.

15. Let M represent snow on Monday and let T represent snow on Tuesday.

$P(M) = 60\%$
$P(T) = 40\%$
$P(M \cap T) = 20\%$
$P(M \cup T) = P(M) + P(T) - P(M \cap T)$
$P(M \cup T) = 60\% + 40\% - 20\%$
$P(M \cup T) = 80\%$

The probability that it will snow on Monday or on Tuesday is 80%.

16. Let S represent damage to the computer's power supply and let C represent damage to other components.

$P(S) = 0.15\%$
$P(C) = 0.30\%$
$P(S \cap C) = 0.10\%$
$P(S \cup C) = P(S) + P(C) - P(S \cap C)$
$P(S \cup C) = 0.15\% + 0.30\% - 0.10\%$
$P(S \cup C) = 0.35\%$

No. e.g., Since the probability of any form of damage is 0.35%, the computer does not need a surge protector.

17. Let the following variables represent the following blood types:

OP: type O+
ON: type O–
AP: type A+
AN: type A–
BP: type B+
BN: type B–
ABP: type AB+
ABN: type AB–

a) i) $P(\text{type O}) = P(OP) + P(ON)$
$P(\text{type O}) = 38\% + 7\%$
$P(\text{type O}) = 45\%$

The probability that a randomly selected Canadian has type O blood is 45%.

ii) $P(\text{negative}) = P(ON) + P(AN) + P(BN) + P(ABN)$
$P(\text{negative}) = 7\% + 6\% + 2\% + 1\%$
$P(\text{negative}) = 16\%$

The probability that a randomly selected Canadian has a negative blood type is 16%.

iii) $P(\text{A or B}) = P(AP) + P(AN) + P(BP) + P(BN)$
$P(\text{A or B}) = 34\% + 6\% + 9\% + 2\%$
$P(\text{A or B}) = 51\%$

The probability that a randomly selected Canadian has type A or B blood is 51%.

b) $P(\text{Dani}) = P(AP) + P(AN) + P(ABP) + P(ABN)$
$P(\text{Dani}) = 34\% + 6\% + 3\% + 1\%$
$P(\text{Dani}) = 44\%$

The probability that Dani can donate blood to the next person who needs a transfusion is 44%.

c) $P(\text{Richard}) = P(ABN) + P(AN) + P(BN) + P(ON)$
$P(\text{Richard}) = 1\% + 6\% + 2\% + 7\%$
$P(\text{Richard}) = 16\%$

The probability that Richard will be able to receive blood is 16%.

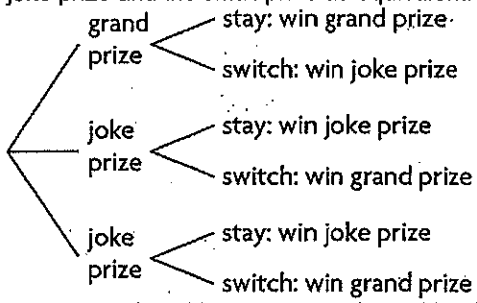
18. e.g., To determine the probability of two events that are not mutually exclusive, you must subtract the probability of both events occurring after adding the probabilities of each event. Example: Female students at a high school may play hockey or soccer. If the probability of a female student playing soccer is 62%, the probability of her playing in goal is 4%, and the probability of her either playing soccer or in goal is 64%, then the probability of her playing in goal at soccer is 62% + 4% – 64% = 2%.

19. A: student who likes rap music
B: student who likes blues music
C: student who likes rock music
O: all students
$P(A \setminus (B \cup C)) = 30\%$
$P(B \setminus (A \cup C)) = 13\%$
$P(C \setminus (A \cup B)) = 20\%$
$P((A \cap C) \setminus B) = 14\%$
$P((B \cap C) \setminus A) = 10\%$
The assumption is made that 100% of students like one of the three genres of music given.
$P(O) = P(A \setminus (B \cup C)) + P(B \setminus (A \cup C)) + P(C \setminus (A \cup B))$
$+ P((A \cap B) \setminus C) + P((A \cap C) \setminus B)$
$+ P((B \cap C) \setminus A) + P(A \cap B \cap C)$
$100\% = 30\% + 13\% + 20\% + P((A \cap B) \setminus C) + 14\%$
$+ 10\% + P(A \cap B \cap C)$
$100\% = 87\% + P((A \cap B) \setminus C) + P(A \cap B \cap C)$
$13\% = P((A \cap B) \setminus C) + P(A \cap B \cap C)$
The probability that a randomly selected student will like either all three types of music, or will like rap and blues, but not rock is 13%.
20. a) $P(A \cup B \cup C) = P(A) + P(B) + P(C)$
b) $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$.

Applying Problem-Solving Strategies, page 181

A. e.g., I don't believe that it will change the probability, because nothing really changes.

B. There's a $\frac{2}{3}$ chance of choosing a joke prize, and a $\frac{1}{3}$ chance of choosing the grand prize. Treating the joke prize and the small prize as equivalent:



After you select, Monty opens a door with a joke prize. If you originally selected the grand prize, and you switch, you will lose. But if you originally selected one of the two joke prizes, and you switch, you will win. So, if you switch, you will win $\frac{2}{3}$ of the time. The better strategy is to switch.

Lesson 3.5: Conditional Probability, page 188

1. a) These two events are dependent.
b) Let R represent the red die showing 4, let S represent rolling a sum that is greater than 7.

$$P(R) = \frac{1}{6}$$
$$P(S|R) = \frac{1}{2}$$
$$P(S \cap R) = P(R) \cdot P(S|R)$$
$$P(S \cap R) = \frac{1}{6} \cdot \frac{1}{2}$$
$$P(S \cap R) = \frac{1}{12}$$

The probability that Austin will win a point is $\frac{1}{12}$, or about 0.0833 or 8.33%.
2. a) These two events are dependent.
b) Let A represent the first card being a diamond, and let B represent the second card being a diamond.

$$P(A) = \frac{13}{52} \qquad P(A \cap B) = P(A) \cdot P(B|A)$$
$$P(A) = \frac{1}{4} \qquad P(A \cap B) = \frac{1}{4} \cdot \frac{4}{17}$$
$$P(B|A) = \frac{12}{51} \qquad P(A \cap B) = \frac{1}{17}$$
$$P(B|A) = \frac{4}{17}$$

The probability that both cards are diamonds is $\frac{1}{17}$, or about 0.0588 or 5.88%.
3. a) These two events are independent.
b) Let D represent drawing a diamond.

$$P(D) = \frac{1}{4}$$
$$P(D \cap D) = \frac{1}{4} \cdot \frac{1}{4}$$
$$P(D \cap D) = \frac{1}{16}$$

The probability that both cards are diamonds is $\frac{1}{16}$, 0.0625 or 6.25%.
4. a) i) Let B represent Lexie pulling a black sock from her drawer.

$$P(B) = \frac{6}{14} \qquad P(B \cap B) = P(B) \cdot P(B|B)$$
$$P(B) = \frac{3}{7} \qquad P(B \cap B) = \frac{3}{7} \cdot \frac{5}{13}$$
$$P(B|B) = \frac{5}{13} \qquad P(B \cap B) = \frac{15}{91}$$

The probability of drawing two black socks is $\frac{15}{91}$, or about 0.165 or 16.5%.

ii) Let W represent Lexie pulling a white sock from her drawer.

$P(W) = \frac{8}{14}$ $P(W \cap W) = P(W) \cdot P(W|W)$

$P(W) = \frac{4}{7}$ $P(W \cap W) = \frac{4}{7} \cdot \frac{7}{13}$

$P(W|W) = \frac{7}{13}$ $P(W \cap W) = \frac{4}{13}$

The probability of drawing two white socks is $\frac{4}{13}$, or about 0.308 or 30.8%.

iii) Let B represent pulling a black sock, and let W represent pulling a white sock. Let A represent drawing a pair of socks.

$P(A) = P(B \cap B) + P(W \cap W)$

$P(A) = \frac{15}{91} + \frac{4}{13}$

$P(A) = \frac{43}{91}$

The probability of drawing a pair of socks is $\frac{43}{91}$, or about 0.473 or 47.3%.

b) No, the answers would not change, because there is still no replacement.

5. a) Let A represent a student who plans to attend UBC, and let O represent all graduating students

$P(A) = \frac{n(A)}{n(O)}$

$P(A) = \frac{30+50}{80+110}$

$P(A) = \frac{80}{190}$

$P(A) = \frac{8}{19}$

The probability that a graduating student will attend UBC is $\frac{8}{19}$, or about 0.421 or 42.1%.

b) Let A represent a student who plans to attend UBC, and let F represent a female student.

$P(F|A) = \frac{n(F)}{n(A)}$

$P(F|A) = \frac{50}{80}$

$P(F|A) = \frac{5}{8}$

The probability this student is female is $\frac{5}{8}$, 0.625 or 62.5%.

6. Let T represent a true or false question, and let M represent a multiple-choice question. Let C represent a correct question.

$P(T \cap C) = P(T) \cdot P(C|T)$ $P(C) = 0.18 + 0.56$

$P(T \cap C) = 0.30 \cdot 0.60$ $P(C) = 0.74$

$P(T \cap C) = 0.18$ $P(M|C) = \frac{P(M \cap C)}{P(C)}$

$P(M \cap C) = P(M) \cdot P(C|M)$

$P(M \cap C) = 0.70 \cdot 0.80$ $P(M|C) = \frac{0.56}{0.74}$

$P(M \cap C) = 0.56$

$P(M|C) = \frac{28}{37}$

The probability the question was multiple-choice is $\frac{28}{37}$, or about 0.757 or 75.7%.

7. Let F represent drawing a loonie the first time, and let S represent drawing a loonie the second time.

$P(F) = \frac{4}{12}$ $P(F \cap S) = P(F) \cdot P(S|F)$

$P(F) = \frac{1}{3}$ $P(F \cap S) = \frac{1}{3} \cdot \frac{3}{11}$

$P(S|F) = \frac{3}{11}$ $P(F \cap S) = \frac{1}{11}$

The probability both coins are loonies is $\frac{1}{11}$, or about 0.091 or 9.1%.

8. Let S represent Anita remembering to set her alarm, and let N represent Anita not remembering to set her alarm. Let L represent Anita being late for school.

$P(S \cap L) = P(S) \cdot P(L|S)$ $P(C) = 0.124 + 0.266$

$P(S \cap L) = 0.62 \cdot 0.20$ $P(C) = 0.39$

$P(S \cap L) = 0.124$ $P(S|L) = \frac{P(S \cap L)}{P(L)}$

$P(N \cap L) = P(N) \cdot P(L|N)$

$P(N \cap L) = 0.38 \cdot 0.70$ $P(M|C) = \frac{0.124}{0.39}$

$P(N \cap L) = 0.266$

$P(M|C) = \frac{62}{195}$

The probability Anita's alarm clock was set is $\frac{62}{195}$, or about 0.318 or 31.8%.

9. Let N represent a nice day, and let R represent a rainy day. Let J represent Ian jogging 8 km.

$P(N \cap J) = P(N) \cdot P(J \mid N)$

$P(N \cap J) = 0.70 \cdot 0.85$

$P(N \cap J) = 0.595$

$P(R \cap J) = P(R) \cdot P(J \mid R)$

$P(R \cap J) = 0.30 \cdot 0.40$

$P(R \cap J) = 0.12$

$P(J) = 0.595 + 0.12$

$P(J) = 0.715$

The probability Ian will jog for 8 km tomorrow is 0.715, or 71.5%.

10. Let C represent a user having call display, and let D represent a user having a data plan.

$P(C \cap D) = P(D) \cdot P(C|D)$

$P(C \cap D) = 0.40 \cdot 0.75$

$P(C \cap D) = 0.30$

$P(D|C) = \frac{P(D \cap C)}{P(C)}$

$P(D|C) = \frac{0.30}{0.70}$

$P(D|C) = \frac{3}{7}$

The probability that a cell phone user with call display also has a data plan is $\frac{3}{7}$, or about 0.429 or 42.9%.

11. e.g., A student selected at random goes to a fast-food outlet that particular day. What is the probability that the student had more than 1 h for lunch?

12. e.g.,

a) Survey questions for a group of classmates over one month (weekdays only):

How often do you cycle to school?	140
How often do you get to school other than by cycling?	60
How often do you cycle when the weather is fine?	80
How often do you cycle when it is raining or snowing?	60

b) A randomly selected student cycled to school on a particular day. What is the probability that the weather was fine that day?

13. Let F represent tires lasting 5 years, and let S represent tires lasting 6 years. Since all tires that have lasted 6 years have also lasted 5 years, it is true that $P(F \cap S) = P(S)$.

$P(S|F) = \frac{P(F \cap S)}{P(F)}$

$P(S|F) = \frac{P(S)}{P(F)}$

$P(S|F) = \frac{0.5}{0.8}$

$P(S|F) = 0.625$

The probability that tires that have lasted 5 years will last 6 years is 0.625, or 62.5%.

14. Let T represent windshield wipers lasting 3 years, and let F represent windshield wipers lasting 4 years. Since all windshield wipers that have lasted 3 years have also lasted 4 years, $P(T \cap F) = P(F)$.

$P(F|T) = \frac{P(T \cap F)}{P(T)}$

$P(F|T) = \frac{P(F)}{P(T)}$

$P(F|T) = \frac{0.6}{0.7}$

$P(F|T) = \frac{6}{7}$

The probability that windshield wipers that have lasted 3 years will last 4 years is $\frac{6}{7}$, or about 0.857 or 85.7%.

15. Let S represent badminton shoes lasting six months, and let Y represent badminton shoes lasting one year. Since all badminton shoes that have lasted one year have also lasted 6 months, $P(S \cap Y) = P(Y)$

$P(Y|S) = \frac{P(S \cap Y)}{P(S)}$

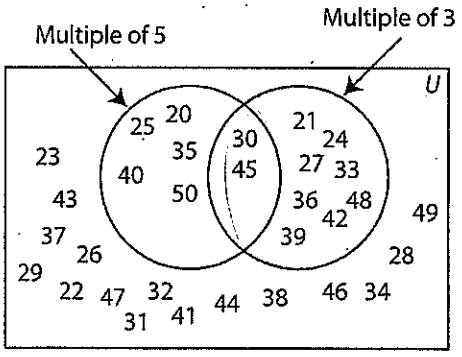
$P(Y|S) = \frac{P(Y)}{P(S)}$

$P(Y|S) = \frac{0.2}{0.9}$

$P(Y|S) = \frac{2}{9}$

The probability that badminton shoes that have lasted six months will last one year is $\frac{2}{9}$, or about 0.222 or 22.2%.

16. a) The probability that a multiple of 5 will also be a multiple of 3 is $\frac{2}{7}$, or about 0.286 or 28.6%.



b) Let F represent a number being a multiple of 5, let T represent a number being a multiple of 3.
F: {20, 25, 30, 35, 40, 45, 50}
T: {21. 24. 27. 30, 33, 36, 39, 42, 45, 48}
30 and 45 are in both sets.

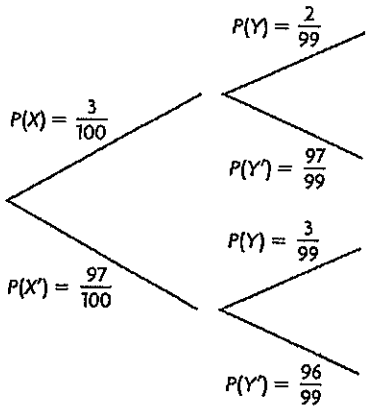
$$P(T|F) = \frac{P(T \cap F)}{P(F)}$$

$$P(T|F) = \frac{\left(\frac{2}{31}\right)}{\left(\frac{7}{31}\right)}$$

$$P(T|F) = \frac{2}{7}$$

The probability that a multiple of 5 will also be a multiple of 3 is $\frac{2}{7}$, or about 0.286 or 28.6%.

17. a) Let X = {1st chip is defective} and Y = {2nd chip is defective}.



b) Let D represent a chip being defective, and let N represent a chip not being defective. Let O represent exactly one of the two chips being defective.
There are two ways to draw two chips, with one being defective. Either the first is defective, or the second is defective. Therefore, the probability of both these events must be determined and added together, to get the total probability.

$$P(D) = \frac{3}{100} \qquad P(N) = \frac{97}{100}$$

$$P(N|D) = \frac{97}{99} \qquad P(D|N) = \frac{3}{99}$$

$$P(D \cap N) = P(D) \cdot P(N|D) \qquad P(D|N) = \frac{1}{33}$$

$$P(D \cap N) = \frac{3}{100} \cdot \frac{97}{99} \qquad P(N \cap D) = P(N) \cdot P(D|N)$$

$$P(D \cap N) = \frac{291}{9900} \qquad P(N \cap D) = \frac{97}{100} \cdot \frac{1}{33}$$

$$P(D \cap N) = \frac{97}{3300} \qquad P(N \cap D) = \frac{97}{3300}$$

Now determine the total probability of drawing exactly one defective chip.

$$P(O) = P(D \cap N) + P(N \cap D)$$

$$P(O) = \frac{97}{3300} + \frac{97}{3300}$$

$$P(O) = \frac{97}{1650}$$

The probability of drawing exactly one defective chip is $\frac{97}{1650}$, or about 0.0588 or 5.88%.

e.g., Multiply the probability of the first chip being defective by the second chip being non-defective, then multiply the probability of the first chip being non-defective by the second chip being defective, and then add the products.

18. Let D represent a chip being defective, and let N represent a chip not being defective. Let O represent exactly one of the chips being defective.
a) There is only one way to draw two defective chips.

$$P(D) = \frac{3}{150} \qquad P(D|D) = \frac{2}{149}$$

$$P(D \cap D) = P(D) \cdot P(D|D)$$

$$P(D \cap D) = \frac{3}{150} \cdot \frac{2}{149}$$

$$P(D \cap D) = \frac{6}{22\,350}$$

$$P(D \cap D) = \frac{1}{3725}$$

The probability of drawing two defective chips is $\frac{1}{3725}$ or about 0.000 268.

b) There is only one way to draw two non-defective chips.

$$P(N) = \frac{147}{150} \qquad P(N|N) = \frac{146}{149}$$

$$P(N \cap N) = P(N) \cdot P(N|N)$$

$$P(N \cap N) = \frac{147}{150} \cdot \frac{146}{149}$$

$$P(N \cap N) = \frac{21\,462}{22\,350}$$

$$P(N \cap N) = \frac{3577}{3725}$$

The probability of drawing two non-defective chips is $\frac{3577}{3725}$ or about 0.960.

c) In this case, either one defective chip is drawn first and then one non-defective chip is drawn, or vice versa. The probability of drawing exactly one defective chip is the sum of the probabilities of these events.

$$P(D)=\frac{3}{150} \qquad P(N)=\frac{147}{150}$$

$$P(N|D)=\frac{147}{149} \qquad P(D|N)=\frac{3}{149}$$

$$P(D\cap N)=P(D)\cdot P(N|D) \qquad P(D\cap N)=P(N)\cdot P(D|N)$$

$$P(D\cap N)=\frac{3}{150}\cdot\frac{147}{149} \qquad P(D\cap N)=\frac{147}{150}\cdot\frac{3}{149}$$

$$P(D\cap N)=\frac{441}{22\,350} \qquad P(D\cap N)=\frac{441}{22\,350}$$

$$P(D\cap N)+P(D\cap N)=\frac{441}{22\,350}+\frac{441}{22\,350}$$

$$P(D\cap N)+P(D\cap N)=\frac{882}{22\,350}$$

$$P(D\cap N)+P(D\cap N)=\frac{147}{3725}$$

The probability of drawing exactly one defective chip is $\frac{147}{3725}$, or about 0.0395.

19. Let S represent a sunny day, and R represent a rainy day. Let W represent a win, and L represent a loss.

a) $P(R \cap W) = P(R) \cdot P(W \mid R)$
$P(R \cap W) = 0.30 \cdot 0.50$
$P(R \cap W) = 0.15$
$P(S \cap W) = P(S) \cdot P(W \mid S)$
$P(S \cap W) = 0.70 \cdot 0.60$
$P(S \cap W) = 0.42$
$P(W) = P(R \cap W) + P(S \cap W)$
$P(W) = 0.15 + 0.42$
$P(W) = 0.57$

The probability Savannah's team will win is 0.57, or 57%.

b) $P(R \cap L) = P(R) \cdot P(L \mid R)$
$P(R \cap L) = 0.30 \cdot 0.50$
$P(R \cap L) = 0.15$
$P(S \cap L) = P(S) \cdot P(L \mid S)$
$P(S \cap L) = 0.70 \cdot 0.40$
$P(S \cap L) = 0.28$
$P(L) = P(R \cap L) + P(S \cap L)$
$P(L) = 0.15 + 0.28$
$P(L) = 0.43$

The probability Savannah's team will lose is 0.43, or 43%.

20. e.g., Problem 1: On weekdays I have cereal for breakfast 70% of the time. On the weekends I have cereal for breakfast 40% of the time. On a random day, what is the probability that I do not have cereal?

Solution:
Let WD represent a weekday, and WE represent a weekend. Let C represent having cereal for breakfast, and let N represent not having cereal for breakfast.

$P(C \mid WD) = 0.70$ means $P(N \mid WD) = 0.30$
$P(C \mid WE) = 0.40$ means $P(N \mid WE) = 0.60$

$$P(N)=\frac{5}{7}\cdot P(N|WD)+\frac{2}{7}\cdot P(N|WE)$$

$$P(N)=\frac{5}{7}\cdot 0.30+\frac{2}{7}\cdot 0.60$$

$$P(N)=\frac{50}{70}\cdot\frac{21}{70}+\frac{20}{70}\cdot\frac{42}{70}$$

$$P(N)=\frac{1050}{4900}+\frac{840}{4900}$$

$$P(N)=\frac{1890}{4900}$$

$$P(N)=\frac{27}{70}$$

Answer: $\frac{27}{70}$ or about 0.386

Problem 2: I draw two cards from a well-shuffled standard deck, drawing the second card without replacing the first one. If my second card is a red card, what is the probability that my first card is black?
Solution: Let B represent the first card being black and R represent the second card being red. We are looking for $P(B \mid R)$.

$$P(B|R)=\frac{P(B\cap R)}{P(R)}$$

$$P(B|R)=\frac{\frac{26}{52}\cdot\frac{26}{51}}{\frac{26}{52}}$$

$$P(B|R)=\frac{26}{51}$$

Answer: $\frac{26}{51}$ or about 0.510

21. e.g., The probability of event A and B both occurring is the probability A occurs, multiplied by the probability B occurs given that A occurs. Example: If a 6-sided die is rolled twice,
P(rolling a 4 the first time and a total > 7)
= P(rolling a 4 the first time)
· P(a total > 7 | rolling a 4 the first time)
or $\frac{1}{12}=\frac{1}{6}\cdot\frac{1}{2}$.

22. Let D represent a chip being defective, and let N represent a chip not being defective.
a) Let S represent the second chip being defective, given that the first chip drawn was also defective. Let T represent the third chip being defective, given that the first two were also defective. Let O represent all three chips drawn being defective. In this case, there is only one way to draw three defective chips.

$P(D)=\frac{4}{100}$ $\quad P(T)=\frac{2}{98}$

$P(D)=\frac{1}{25}$ $\quad P(T)=\frac{1}{49}$

$P(S)=\frac{3}{99}$ $\quad P(O)=P(D)\cdot P(S)\cdot P(T)$

$P(S)=\frac{1}{33}$ $\quad P(O)=\frac{1}{25}\cdot\frac{1}{33}\cdot\frac{1}{49}$

$P(O)=\frac{1}{40425}$

The probability of drawing 3 defective chips is $\frac{1}{40\ 425}$, or about 0.000 024 7 or 0.002 47%.
b) Let B represent the second chip not being defective, given that the first chip drawn was also not defective. Let C represent the third chip not being defective, given that the first two were also not defective. Let F represent all three chips drawn not being defective. In this case, there is only one way to draw three non-defective chips.

$P(N)=\frac{96}{100}$ $\quad P(B)=\frac{95}{99}$ $\quad P(C)=\frac{94}{98}$

$P(N)=\frac{24}{25}$ $\quad P(C)=\frac{47}{49}$

$P(F)=P(N)\cdot P(B)\cdot P(C)$

$P(F)=\frac{24}{25}\cdot\frac{95}{99}\cdot\frac{47}{49}$

$P(F)=\frac{107160}{121275}$

$P(F)=\frac{7144}{8085}$

The probability of drawing 3 defective chips is $\frac{7144}{8085}$, or about 0.884 or 88.4%.
c) In this case, more defective chips will be pulled when 2 or 3 defective chips are pulled. There are 3 different ways to pull 2 defective chips.
Let S represent the second chip being defective, given that the first chip drawn was also defective. Let U represent the third chip not being defective, given that the first two were defective. Let G represent two of the three chips drawn being defective. Let A represent at least two of the three chips drawn being defective.

$P(D)=\frac{4}{100}$ $\quad P(S)=\frac{3}{99}$ $\quad P(U)=\frac{96}{98}$

$P(D)=\frac{1}{25}$ $\quad P(S)=\frac{1}{33}$ $\quad P(U)=\frac{48}{49}$

$P(G)=P(D)\cdot P(S)\cdot P(U)\cdot 3$

$P(G)=\frac{1}{25}\cdot\frac{1}{33}\cdot\frac{48}{49}\cdot 3$

$P(G)=\frac{144}{40425}$

$P(G)=\frac{48}{13475}$

The total probability can now be determined.
$P(A)=P(G)+P(O)$

$P(A)=\frac{48}{13475}+\frac{1}{40425}$

$P(A)=\frac{144}{40425}+\frac{1}{40425}$

$P(A)=\frac{145}{40425}$

$P(A)=\frac{29}{8085}$

The probability of drawing more defective chips than non-defective chips is $\frac{29}{8085}$, or about 0.0036 or 0.36%.

Lesson 3.6: Independent Events, page 198

1. a) These events are independent, because the result of the spinner does not affect the result of the die, and vice versa.
b) These events are independent, because the result of the red die does not affect the result of the green die, and vice versa.
c) These events are dependent. Because there is no replacement, the deck that the second card is drawn from is technically different than the original deck.
d) These events are independent, because replacement is occurring, which 'resets' the probability for each draw.

2. a) These events are likely independent. The cardio workouts focus on the heart, and use the legs the most often. Therefore, there would be no specific reason why one workout would be favoured over another.
b) Let B represent Celeste using a stationary bike, and let F represent Celeste using free weights.

$P(B)=\frac{1}{3}$ $\quad P(F)=\frac{1}{2}$

$P(B\cap F)=P(B)\cdot P(F)$

$P(B\cap F)=\frac{1}{3}\cdot\frac{1}{2}$

$P(B\cap F)=\frac{1}{6}$

The probability Celeste will use a stationary bike and free weights for her next workout is $\frac{1}{6}$, or about 0.167 or 16.7%.

3. a) The two workouts are dependent. After Ian chooses a workout, he won't choose the same one again.

b) Let T represent Ian running the track, and let E represent Ian using an elliptical walker.

$$P(T) = \frac{1}{4} \qquad P(E|T) = \frac{1}{3}$$

$$P(T \cap E) = P(T) \cdot P(E|T)$$

$$P(T \cap E) = \frac{1}{4} \cdot \frac{1}{3}$$

$$P(T \cap E) = \frac{1}{12}$$

The probability that Ian will run the track and use the elliptical walker is $\frac{1}{12}$, or about 0.0833 or 8.33%.

4. a) Let R represent spinning a red, and let T represent rolling a two.

$$P(R) = \frac{1}{4} \qquad P(T) = \frac{1}{6}$$

$$P(R \cap T) = P(R) \cdot P(T)$$

$$P(R \cap T) = \frac{1}{4} \cdot \frac{1}{6}$$

$$P(R \cap T) = \frac{1}{24}$$

The probability the spinner will land on red and the die will land on 2 is $\frac{1}{24}$, or about 0.0417 or 4.17%.

b) Let O represent rolling a one on the red die, and F represent rolling a five on the green die.

$$P(O) = \frac{1}{6} \qquad P(F) = \frac{1}{6}$$

$$P(O \cap F) = P(O) \cdot P(F)$$

$$P(O \cap F) = \frac{1}{6} \cdot \frac{1}{6}$$

$$P(O \cap F) = \frac{1}{36}$$

The probability of rolling a 1 on the red die and a 5 on the green die is $\frac{1}{36}$, or about 0.0278 or 2.78%.

c) Let K represent drawing a king, and let A represent drawing an ace.

$$P(K) = \frac{4}{52}$$

$$P(K) = \frac{1}{13}$$

$$P(A|K) = \frac{4}{51}$$

$$P(K \cap A) = P(K) \cdot P(A|K)$$

$$P(K \cap A) = \frac{1}{13} \cdot \frac{4}{51}$$

$$P(K \cap A) = \frac{4}{663}$$

The probability that a king will be drawn first, and an ace will be drawn second is $\frac{4}{663}$, or about 0.006 or 0.6%.

d) Let N represent drawing a prime number, and let F represent drawing a multiple of five.

$$P(N) = \frac{10}{30} \qquad P(F) = \frac{6}{30}$$

$$P(N) = \frac{1}{3} \qquad P(F) = \frac{1}{5}$$

$$P(N \cap F) = P(N) \cdot P(F)$$

$$P(N \cap F) = \frac{1}{3} \cdot \frac{1}{5}$$

$$P(N \cap F) = \frac{1}{15}$$

The probability the first card is prime and the second card is a multiple of 5 is $\frac{1}{15}$, or about 0.0667 or 6.67%.

5. a) If two events are independent, then $P(A \cap B)$ is equal to $P(A) \cdot P(B)$.

$P(A \cap B) = 0.12$

$P(A) \cdot P(B) = 0.35 \cdot 0.4$

$P(A) \cdot P(B) = 0.14$

Since $P(A \cap B) \neq P(A) \cdot P(B)$, no, these events are not independent.

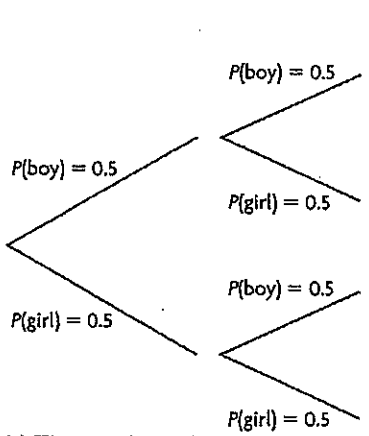
b) $P(A \cap B) = 0.468$

$P(A) \cdot P(B) = 0.720 \cdot 0.650$

$P(A) \cdot P(B) = 0.468$

Since $P(A \cap B) = P(A) \cdot P(B)$, yes, these events are independent.

6. a)



b) The genders of the children are independent events. Also, there is only one way that both children can be boys. Let B represent a child being a boy.

$P(B) = 0.5$

$P(B \cap B) = P(B) \cdot P(B)$

$P(B \cap B) = 0.5 \cdot 0.5$

$P(B \cap B) = 0.25$

The probability both children are boys is 0.25, or 25%.

c) There are two ways that one child can be a boy and the other can be a girl.
Let B represent a child being a boy, and let G represent a child being a girl.

$P(B) = 0.5$
$P(G) = 0.5$
$P(B \cap G) = P(B) \cdot P(G) \cdot 2$
$P(B \cap G) = 0.5 \cdot 0.5 \cdot 2$
$P(B \cap G) = 0.5$

The probability that one child is a boy and the other is a girl is 0.5, or 50%.

7. These events are dependent, because the deck has 40 cards when the first card is dealt, but it has 39 cards when the second card is dealt.
Let C represent a club being dealt, and let H represent a heart being dealt.

$P(C) = \frac{10}{40}$ $\quad P(C \cap H) = P(C) \cdot P(H|C)$

$P(C) = \frac{1}{4}$ $\quad P(C \cap H) = \frac{1}{4} \cdot \frac{10}{39}$

$P(H|C) = \frac{10}{39}$ $\quad P(C \cap H) = \frac{10}{156}$

$P(C \cap H) = \frac{5}{78}$

The probability the first card dealt is a club and the second card dealt is a heart is $\frac{5}{78}$, or about 0.0641 or 6.41%.

8. Let A represent the first roll, and B represent the second.

a) $P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = \frac{1}{6} \cdot \frac{1}{6}$

$P(A \cap B) = \frac{1}{36}$

The probability is $\frac{1}{36}$.

b) $P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = \frac{3}{6} \cdot \frac{3}{6}$

$P(A \cap B) = \frac{1}{2} \cdot \frac{1}{2}$

$P(A \cap B) = \frac{1}{4}$

The probability is $\frac{1}{4}$.

c) $P(A \cap B) = P(A) \cdot P(B)$

$P(A \cap B) = \frac{5}{6} \cdot \frac{5}{6}$

$P(A \cap B) = \frac{25}{36}$

The probability is $\frac{25}{36}$.

9. These two events are independent. Let C represent seeing a camel, and let B represent seeing an ibis.

a) $P(C \cap B) = P(C) \cdot P(B)$

$P(C \cap B) = \frac{4}{5} \cdot \frac{3}{4}$

$P(C \cap B) = \frac{3}{5}$

The probability Jeremiah will see a camel and an ibis is $\frac{3}{5}$, 0.6 or 60%.

b) $P(C') = 1 - P(C)$ $\quad P(B') = 1 - P(B)$

$P(C') = 1 - \frac{4}{5}$ $\quad P(B') = 1 - \frac{3}{4}$

$P(C') = \frac{1}{5}$ $\quad P(B') = \frac{1}{4}$

$P(C' \cap B') = P(C') \cdot P(B')$

$P(C' \cap B') = \frac{1}{5} \cdot \frac{1}{4}$

$P(C' \cap B') = \frac{1}{20}$

The probability Jeremiah will see neither animal is $\frac{1}{20}$, 0.05 or 5%.

c) The probability Jeremiah will see only one of these animals is the same as the probability that he will not see both or neither of them. So, subtract these probabilities from 1 to determine the new probability.
Let O represent Jeremiah seeing only one of these animals.

$P(O) = 1 - P(C \cap B) - P(C' \cap B')$

$P(O) = 1 - \frac{3}{5} - \frac{1}{20}$

$P(O) = \frac{7}{20}$

The probability Jeremiah will see only one of these animals is $\frac{7}{20}$, 0.35 or 35%.

10. a) These two events are independent. Let H represent getting heads from tossing a coin, and let S represent spinning a six on a spinner.

$P(H) \cdot P(S) = P(H \cap S)$

$\frac{1}{2} \cdot P(S) = \frac{1}{12}$

$\frac{2}{2} \cdot P(S) = \frac{1}{12} \cdot 2$

$P(S) = \frac{1}{6}$

The probability of spinning a six is $\frac{1}{6}$.

e.g., Spinner has 6 equal areas, numbered 1 to 6.

b) These two events are independent. Let H represent getting heads from tossing a coin, and let S represent spinning a six on a spinner.

$P(H) \cdot P(S) = P(H \cap S)$

$\frac{1}{2} \cdot P(S) = \frac{1}{20}$

$\frac{2}{2} \cdot P(S) = \frac{1}{20} \cdot 2$

$P(S) = \frac{1}{10}$

The probability of spinning a six is $\frac{1}{10}$.

e.g., Spinner has 10 equal areas, numbered 1 to 10.

11. a) Anne's first bag contains 7 red marbles and 3 blue marbles, and her second bag contains 4 red marbles and 5 blue marbles. Because she is drawing 2 marbles from 2 separate bags, the two events are independent.

Let F represent a blue marble drawn from the first bag, and let S represent a blue marble drawn from the second bag. Let B represent drawing two blue marbles.

$P(F) = \frac{3}{10}$ $\quad P(S) = \frac{5}{9}$

$P(B) = P(F) \cdot P(S)$

$P(B) = \frac{3}{10} \cdot \frac{5}{9}$

$P(B) = \frac{1}{6}$

The probability Anne will draw two blue marbles is $\frac{1}{6}$, or about 0.167 or 16.7%.

Abby's bag contains 11 red marbles and 8 blue marbles. Since she is drawing 2 marbles from the same bag, the two events are dependent.

Let F represent the first marble drawn being blue, and let S represent the second marble drawn being blue. Let B represent drawing two blue marbles.

$P(F) = \frac{8}{19}$ $\quad P(B) = P(F) \cdot P(S|F)$

$P(S|F) = \frac{7}{18}$ $\quad P(B) = \frac{8}{19} \cdot \frac{7}{18}$

$P(B) = \frac{28}{171}$

The probability Abby will draw two blue marbles is $\frac{28}{171}$, or about 0.164 or 16.4%.

No. e.g., Anne has a $\frac{1}{6}$ or about 0.167 probability of drawing two blue marbles, while Abby has a $\frac{28}{171}$ or about 0.164 probability of drawing two blue marbles.

b) For both girls, there are two ways that they can draw one red marble and one blue marble. However, in Abby's case, the probabilities will be the same, because the events are dependent. Therefore, only one probability needs to be determined, and that result can be multiplied by 2 to determine the total probability.

Anne: Let F represent Anne drawing a red marble from her first bag, and a blue marble from her second bag, and let S represent Anne drawing a blue marble from her first bag and a red marble from her second bag. Let O represent drawing one red marble and one blue marble.

$P(F) = \frac{7}{10} \cdot \frac{5}{9}$ $\quad P(S) = \frac{3}{10} \cdot \frac{4}{9}$

$P(F) = \frac{7}{18}$ $\quad P(S) = \frac{2}{15}$

$P(O) = P(F) + P(S)$

$P(O) = \frac{7}{18} + \frac{2}{15}$

$P(O) = \frac{35}{90} + \frac{12}{90}$

$P(O) = \frac{47}{90}$

Abby: Let R represent drawing a red marble, and let B represent drawing a blue marble. Let OA represent drawing one red marble and one blue marble.

$P(R) = \frac{11}{19}$ $\quad P(OA) = 2 \cdot P(R) \cdot P(B|R)$

$P(B|R) = \frac{8}{18}$ $\quad P(OA) = 2 \cdot \frac{11}{19} \cdot \frac{4}{9}$

$P(B|R) = \frac{4}{9}$ $\quad P(OA) = \frac{88}{171}$

Let A represent that Anne and Abby will both draw one red marble and one blue marble.

$P(A) = P(O) \cdot P(OA)$

$P(A) = \frac{47}{90} \cdot \frac{88}{171}$

$P(A \cap B) = \frac{4136}{15\,390}$

$P(A \cap B) = \frac{2068}{7695}$

The probability both girls will each draw one red and one blue marble is $\frac{2068}{7695}$, or about 0.269 or 26.9%.

c) Anne: Let F represent drawing a red marble from the first bag, and let S represent drawing a red marble from the second bag. Let R represent drawing two red marbles.

$P(F) = \frac{5}{10}$ $\quad P(R) = P(F) \cdot P(S)$

$P(F) = \frac{1}{2}$ $\quad P(R) = \frac{1}{2} \cdot \frac{1}{2}$

$P(S) = \frac{1}{2}$ $\quad P(R) = \frac{1}{4}$

The probability Anne will draw two red marbles is $\frac{1}{4}$, 0.25 or 25%.

Abby: Let F represent the first marble drawn being red, and let S represent the second marble drawn being red. Let R represent drawing two red marbles.

$P(F)=\frac{10}{20}$ $\quad P(O)=P(F)\cdot P(S|F)$

$P(F)=\frac{1}{2}$ $\quad P(O)=\frac{1}{2}\cdot\frac{9}{19}$

$P(S|F)=\frac{9}{19}$ $\quad P(O)=\frac{9}{38}$

The probability that Abby draws two red marbles is $\frac{9}{38}$, or about 0.237 or 23.7%.

No, e.g., Anne has a $\frac{1}{4}$ or 0.25 probability of drawing two red marbles, while Abby has a $\frac{9}{38}$ or about 0.237 probability of drawing two red marbles.

12. The two drawings are independent, because the first treat is replaced after it is picked.

Let G represent drawing a granola bar, let F represent drawing a fruit bar, and let C represent drawing a cheese strip.

a) The second granola bar can be kept if a granola bar is drawn both times.

$P(G)=\frac{10}{20}$ $\quad P(G\cap G)=P(G)\cdot P(G)$

$P(G)=\frac{1}{2}$ $\quad P(G\cap G)=\frac{1}{2}\cdot\frac{1}{2}$

$P(G\cap G)=\frac{1}{4}$

The probability a granola bar is kept is $\frac{1}{4}$, 0.25 or 25%.

b) Let K represent keeping a treat. Any treat can be kept if two of the same treats are drawn.

$P(F)=\frac{7}{20}$ $\quad P(C)=\frac{3}{20}$

$P(F\cap F)=P(F)\cdot P(F)$ $\quad P(C\cap C)=P(C)\cdot P(C)$

$P(F\cap F)=\frac{7}{20}\cdot\frac{7}{20}$ $\quad P(C\cap C)=\frac{9}{400}$

$P(K)=P(G\cap G)+P(F\cap F)+P(C\cap C)$

$P(K)=\frac{1}{4}+\frac{49}{400}+\frac{9}{400}$

$P(K)=\frac{79}{200}$

The probability that any treat can be kept is $\frac{79}{200}$, 0.395 or 39.5%.

c) $P(K')=1-P(K)$

$P(K')=1-\frac{79}{200}$

$P(K')=\frac{121}{200}$

The probability a treat is not kept is $\frac{121}{200}$, 0.605 or 60.5%.

13. Because the winner's ticket is returned to the draw after the first prize is awarded, the two events are independent.

Let W represent Tiegan winning the first prize, and let X represent winning the second prize.

a) $P(W)=\frac{5}{100}$ $\quad P(W\cap X)=P(W)\cdot P(X)$

$P(W)=\frac{1}{20}$ $\quad P(W\cap X)=\frac{1}{20}\cdot\frac{1}{20}$

$P(X)=\frac{1}{20}$ $\quad P(W\cap X)=\frac{1}{400}$

The probability Tiegan wins both prizes is $\frac{1}{400}$, 0.0025 or 0.25%.

b) $P(W')=1-P(W)$ $\quad P(W'\cap X')=P(W')\cdot P(X')$

$P(W')=1-\frac{1}{20}$ $\quad P(W'\cap X')=\frac{19}{20}\cdot\frac{19}{20}$

$P(W')=\frac{19}{20}$ $\quad P(W'\cap X')=\frac{361}{400}$

$P(W')=P(X')$

The probability that Tiegan wins neither prize is $\frac{361}{400}$, 0.9025 or 90.25%.

14. a) e.g., Problem: What is the probability of drawing a card from a shuffled standard deck and getting a red card, then replacing it, shuffling the deck again, drawing a second card, and getting a heart?

Solution: Let R represent getting a red card and H represent getting a heart.

$P(R\cap H)=P(R)\cdot P(H)$

$P(R\cap H)=\frac{26}{52}\cdot\frac{13}{52}$

$P(R\cap H)=\frac{1}{2}\cdot\frac{1}{4}$

$P(R\cap H)=\frac{1}{8}$

The probability is $\frac{1}{8}$, 0.125 or 12.5%.

b) e.g., Problem: What is the probability of drawing a card from a shuffled standard deck and getting a red card, then drawing a second card without replacing the first one, and getting a spade?

Solution: Let R represent getting a red card on the first draw, and S represent getting a spade on the second draw.

$$P(R) = \frac{26}{52} \qquad P(S \mid R) = \frac{13}{51}$$

$$P(R \cap S) = P(R) \cdot P(S \mid R)$$

$$P(R \cap S) = \frac{26}{52} \cdot \frac{13}{51}$$

$$P(R \cap S) = \frac{13}{102}$$

The probability is $\frac{13}{102}$, or about 0.127 or 12.7%.

15. There are nine ways that two single-digit numbers can add up to 10, and there are 100 ways to select two single-digit numbers. Therefore, the probability the sum of these two numbers is 10 is $\frac{9}{100}$, 0.09 or 9%.

16. a) The formula is $P(A \cap B) = P(A) \cdot P(B)$ only when A and B are independent events.
b) e.g. Drawing two red marbles from a bag containing 5 red and 15 blue marbles, with replacement:
A = {red on 1st draw} and B = {red on 2nd draw} are independent events, so

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = \frac{1}{4} \cdot \frac{1}{4}$$

$$P(A \cap B) = \frac{1}{16}$$

c) e.g., Drawing two red marbles from a bag containing 5 red and 15 blue marbles, without replacement:
A = {red on 1st draw} and B = {red on 2nd draw} are dependent events so

$$P(A \cap B) = P(A) \cdot P(B \mid A)$$

$$P(A \cap B) = \frac{1}{4} \cdot \frac{4}{19}$$

$$P(A \cap B) = \frac{1}{19}$$

17. The failure of each part is independent. Let F represent a part failing, and let O represent no parts failing for one year.
a) $P(F) = 0.01$
$P(F') = 0.99$
Because each event is independent, the probability that no parts fail is determined by multiplying the probability that one part does not fail by itself 100 times.

$$P(O) = (P(F'))^{100}$$

$$P(O) = (0.99)^{100}$$

$$P(O) = 0.366...$$

The probability that the machine will operate continuously for one year is about 0.366 or 36.6%.

b) $P(F) = 0.005$
$P(F') = 0.995$
Determine the probability that no parts can fail in the same way.

$$P(O) = (P(F'))^{100}$$

$$P(O) = (0.995)^{100}$$

$$P(O) = 0.605...$$

The probability the machine will operate continuously for one year is about 0.606 or 60.6%.

c)

$$P(O) = (P(F'))^{100}$$

$$0.9 = (P(F'))^{100}$$

$$\sqrt[100]{0.9} = \sqrt[100]{(P(F'))^{100}}$$

$$P(F') = 0.998...$$

The probability of not failing needs to be about 0.999 or 99.9% to ensure that the probability that the machine will operate continuously is 90%.

18. Let W represent a win, and let L represent a loss.
a) There is only one way that four games in a row can be wins. Let F represent winning the series in 4 games.
$P(F) = P(W) \cdot P(W) \cdot P(W) \cdot P(W)$
$P(F) = (0.65)^4$
$P(F) = 0.178...$
The probability that Montreal would win the series in four games is about 0.179 or 17.9%.
b) Since the odds in favour of Montreal winning four games in a row were 5 : 4, the probability of this occurring was $\frac{5}{9}$ or about 0.555.... Let F represent winning the series in 4 games.
$P(F) = P(W) \cdot P(W) \cdot P(W) \cdot P(W)$
$P(F) = P(W)^4$
$0.555... = P(W)^4$
Taking the square root twice:
$P(W) = 0.863...$
The probability Montreal would win any single game in the series was about 0.863... or 86.3%, assuming the chance of winning any of the four games was equal.

19. To pass each test, Gavin must answer at least three questions correctly. For the true-false test, the probability of answering each question correctly (C), is 0.5, as is the probability of answering it wrongly (W). By the tree diagram, there are 32 different possible outcomes, each one with an equal chance of occurring (0.031 25). Of these 32 outcomes, there are 16 in which at least 3 answers are correct so the probability of passing the true-false test is, $\frac{16}{32}$ or 0.5.

Note: Another way to calculate this is $16 \cdot 0.03125 = 0.5$.

With the multiple-choice test, the probability of answering a question correctly is $\frac{1}{4}$ and the probability of answering it wrongly is $\frac{3}{4}$. There are also 32 possible outcomes, 16 of which allow for at least 3 correct answers. These outcomes are not equally likely.
One outcome has 5 correct answers.

$P(\text{5 correct, 0 wrong}) = \left(\frac{1}{4}\right)^5$

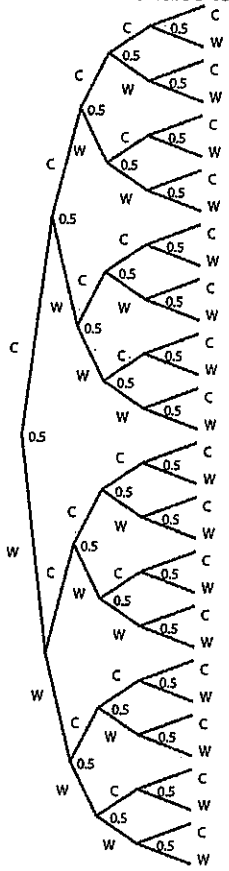
Five outcomes have 4 correct answers and one wrong answer. $5 \cdot P(\text{4 correct, 1 wrong}) = 5\left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right)$

Ten outcomes have 3 correct answers and 2 wrong answers. $10 \cdot P(\text{3 correct, 1 wrong}) = 10\left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^2$

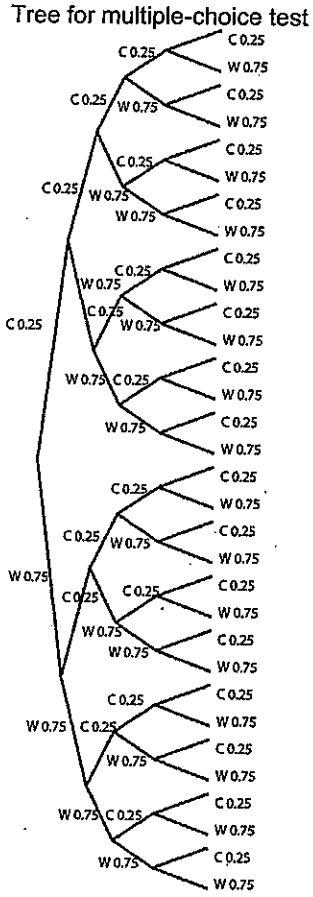
$$P(\text{at least 3 correct answers})$$
$$= \left(\frac{1}{4}\right)^5 + 5\left(\frac{1}{4}\right)^4 \cdot \left(\frac{3}{4}\right) + 10\left(\frac{1}{4}\right)^3 \cdot \left(\frac{3}{4}\right)^2$$
$$= \frac{1}{1024} + \frac{15}{1024} + \frac{90}{1024}$$
$$= \frac{106}{1024}$$
$$= \frac{53}{512} \text{ or } 0.1035...$$

The probability of passing the multiple-choice test is $\frac{53}{512}$ or about 0.1035.

Tree for true-false test



Tree for multiple-choice test



Math in Action, page 201

The group found that the experiment of drawing one ball at a time was the most productive. This is because when two balls are drawn at once, conditional probability exists, which can be hard to deal with.
The best way to make the conjecture more reliable is to increase the number of trials. When this is done, the experimental probability slowly begins to act more like the theoretical probability.
However, there is no type of experiment that can guarantee a correct conjecture. This is because theoretical probability is just that; theoretical. This probability is only guaranteed to work after millions or possibly billions of trials, or continuous trials.

Chapter Self-Test, page 202

1. Outcome Table

		Tile 1					
	SUM	1	2	3	4	5	6
Tile 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

$P(\text{odd sum}) = \frac{18}{36}$ $P(\text{even sum}) = \frac{18}{36}$

$P(\text{odd sum}) = \frac{1}{2}$ $P(\text{even sum}) = \frac{1}{2}$

Yes, e.g., The probabilities of winning are equal.

2. Let A represent an Inuit person being able to converse in at least two Aboriginal languages.

$$P(A) = \frac{7}{7+3}$$

$$P(A) = \frac{7}{10}$$

The probability an Inuit person can converse in at least two Aboriginal languages is $\frac{7}{10}$, 0.7 or 70%.

3. Let A represent a passcode with 3 different even digits, and let O represent all 3-digit passcodes. There are 5 different possibilities for even digits: 0, 2,4,6, and 8. Since these numbers cannot be repeated, the total number of passcodes using 3 different even digits is equal to ${}_5P_3$.

$$n(A) = {}_5P_3$$

$$n(A) = \frac{5!}{(5-3)!}$$

$$n(A) = \frac{5!}{2!}$$

$$n(A) = \frac{5 \cdot 4 \cdot 3 \cdot 2!}{2!}$$

$$n(A) = 5 \cdot 4 \cdot 3$$

$$n(A) = 60$$

The number of 3 digit passcodes is 10^3, or 1000, because there are 10 possible digits to use in 3 spaces, and repeating is allowed.

$$P(A) = \frac{n(A)}{n(O)}$$

$$P(A) = \frac{60}{1000}$$

$$P(A) = \frac{3}{50}$$

The probability that Braydon's passcode is made up of three different even digits is $\frac{3}{50}$, 0.06 or 6%.

4. Let A represent a hand containing 4 hearts and 4 spades, and let O represent all 8-card hands. There are 13 hearts and 13 spades in a standard deck. Therefore, the total number of hands that contain 4 hearts and 4 spades is ${}_{13}C_4 \cdot {}_{13}C_4$, or $({}_{13}C_4)^2$.

$$n(A) = \left({}_{13}C_4\right)^2$$

$$n(A) = \left(\frac{13!}{(13-4)! \cdot 4!}\right)^2$$

$$n(A) = \left(\frac{13!}{9! \cdot 4!}\right)^2$$

$$n(A) = \left(\frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9!}{9! \cdot 4 \cdot 3 \cdot 2 \cdot 1}\right)^2$$

$$n(A) = \left(\frac{13 \cdot 12 \cdot 11 \cdot 10}{4 \cdot 3 \cdot 2}\right)^2$$

$$n(A) = \left(5 \cdot 11 \cdot 13\right)^2$$

$$n(A) = 5^2 \cdot 11^2 \cdot 13^2$$

The total number of 8-card hands is ${}_{52}C_8$.

$$n(O) = {}_{52}C_8$$

$$n(O) = \frac{52!}{(52-8)! \cdot 8!}$$

$$n(O) = \frac{52!}{44! \cdot 8!}$$

$$n(O) = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45 \cdot 44!}{44! \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$n(O) = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45}{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}$$

$$n(O) = \frac{48}{8 \cdot 6} \cdot \frac{49}{7} \cdot \frac{50}{5} \cdot \frac{52}{4} \cdot \frac{51}{3} \cdot \frac{46}{2} \cdot 45 \cdot 47$$

$$n(O) = 7 \cdot 10 \cdot 13 \cdot 17 \cdot 23 \cdot 45 \cdot 47$$

Now determine the probability:

$$P(A) = \frac{n(A)}{n(O)}$$

$$P(A) = \frac{5^2 \cdot 11^2 \cdot 13^2}{7 \cdot 10 \cdot 13 \cdot 17 \cdot 23 \cdot 45 \cdot 47}$$

$$P(A) = \frac{11^2 \cdot 13}{2 \cdot 7 \cdot 9 \cdot 17 \cdot 23 \cdot 47}$$

$$P(A) = \frac{1573}{2\,315\,502}$$

The probability that a hand consists of 4 hearts and 4 spades is $\frac{1573}{2315502}$ or about 0.000 679.

5. $P(\text{truck or yo-yo}) = \frac{10}{50}$

$P(\text{truck or yo-yo}) = \frac{1}{5}$

$P(\text{truck or yo-yo}) = 0.2$

The probability that Kaylee will win either a toy truck or a yo-yo is 0.2, or 20%.

6. These events are independent.
Let A represent Hans flipping a coin on heads, and let B represent Hans drawing an 8.

$P(A)=\frac{1}{2}$ $\quad P(A\cap B)=P(A)\cdot P(B)$

$P(B)=\frac{4}{52}$ $\quad P(A\cap B)=\frac{1}{2}\cdot\frac{1}{13}$

$P(B)=\frac{1}{13}$ $\quad P(A\cap B)=\frac{1}{26}$

The probability that Hans will flip a head and draw an 8 is $\frac{1}{26}$, or about 0.0385 or 3.85%.

7. Let F represent making a free throw.
a) $P(F) = 0.7$
$P(F \cap F) = 0.7 \cdot 0.7$
$P(F \cap F) = 0.49$
The probability Jarrod will make both shots is 0.49, or 49%.
b) $P(F') = 0.3$
$P(F' \cap F') = 0.3 \cdot 0.3$
$P(F' \cap F') = 0.09$
The probability Jarrod will make both shots is 0.09, or 9%.
c) In this case, there are only 3 possibilities. These are make both shots, make one shot, and make neither shot.
Let O represent Jarrod making exactly one shot.
$P(O) = 100\% - P(F \cap F) - P(F' \cap F')$
$P(O) = 100\% - 49\% - 9\%$
$P(O) = 42\%$
The probability Jarrod will make one shot is 0.42 or 42%.
d) Let A represent Jarrod making at least one shot.
$P(A) = 100\% - P(F' \cap F')$
$P(A) = 100\% - 9\%$
$P(A) = 91\%$
The probability Jarrod will make at least one shot is 0.91 or 91%.

8. Let H represent two hearts being drawn from the deck, and let O represent all possible drawings.
There are 6 hearts in every euchre deck. Therefore, the number of ways to draw two hearts from a Euchre deck is ${}_6C_2$.

$n(H)={}_6C_2$

$n(H)=\frac{6!}{(6-2)!\cdot 2!}$

$n(H)=\frac{6!}{4!\cdot 2!}$

$n(H)=\frac{6\cdot 5\cdot 4!}{4!\cdot 2\cdot 1}$

$n(H)=\frac{6\cdot 5}{2}$

$n(H)=15$

The number of ways to draw 2 cards from a Euchre deck is ${}_{24}C_2$.

$n(O)={}_{24}C_2$

$n(O)=\frac{24!}{(24-2)!\cdot 2!}$

$n(O)=\frac{24!}{22!\cdot 2!}$

$n(O)=\frac{24\cdot 23\cdot 22!}{22!\cdot 2\cdot 1}$

$n(O)=\frac{24\cdot 23}{2}$

$n(O)=276$

Now determine the probability.

$P(H)=\frac{n(H)}{n(O)}$

$P(H)=\frac{15}{276}$

$P(H)=\frac{5}{92}$

The probability that George draws two hearts is $\frac{5}{92}$, or about 0.0543 or 5.43%.

9. Let S represent a set alarm clock, and let N represent an alarm clock that is not set. Let L represent Miguel being late.

$P(S\cap L)=P(S)\cdot P(L|S)$

$P(S\cap L)=0.72\cdot 0.10$

$P(S\cap L)=0.072$

$P(N\cap L)=P(N)\cdot P(L|N)$

$P(N\cap L)=0.28\cdot 0.70$

$P(N\cap L)=0.196$

$P(L)=0.072+0.196$

$P(L)=0.268$

$P(S|L)=\frac{P(S\cap L)}{P(L)}$

$P(S|L)=\frac{0.072}{0.268}$

$P(S|L)=26.865...\%$

The probability Miguel remembered to set his alarm is about 0.269 or 26.9%.

Chapter Review, page 205

1. a) Outcome Table

Coin Flips	Winner
HHHH	Chloe
HHHT	Tie
HHTH	Tie
HHTT	Camila
HTHH	Tie
HTHT	Camila
HTTH	Camila
HTTT	Tie
THHH	Tie
THHT	Camila
THTH	Camila
THTT	Tie
TTHH	Camila
TTHT	Tie
TTTH	Tie
TTTT	Chloe

$P(\text{Chloe wins}) = \frac{2}{16}$

$P(\text{Chloe wins}) = \frac{1}{8}$

$P(\text{Camila wins}) = \frac{6}{16}$

$P(\text{Camila wins}) = \frac{3}{8}$

This game is not fair, e.g., Camila has a better chance of winning.

b) It is equally like that a 1 or a 2 will come up than a 3 or a 4. Therefore, the game is fair, Cooper and Alyssa will have an equal chance at winning.

2. The probability is 0, because when the tension is at 100% you can no longer turn the pedal. At this point the tension is at its maximum. Turning the tension up to 130% implies that tension can be increased beyond its maximum setting. What Bob's instructor probably means is to adjust the tension so that it is 1.3 times greater than it currently is, which may be possible.

3. a) $P(\text{female}) = 60\%$

$P(\text{not female}) = 40\%$

The odds in favour of this person being female are 60 : 40, or 3 : 2

b) The odds against this person being female are 2 : 3.

4. The odds against rain tomorrow are (100% – 70%) : 70%. This is equal to 30 : 70, or 3 : 7.

5. a) If Keir fell twice, then he didn't fall the other five times. Therefore, the odds in favour of him falling are 2 : 5.

b) The odds against Keir falling are 5 : 2.

6. No, she is not correct. e.g., To calculate the odds, it should be odds against : odds for. If Ariana has scored 6 goals on 30 shots, then she did not score 24 times. Therefore, the odds against her scoring a goal are 24 : 6, or 4 : 1, not 4 : 5.

7. Let F represent Averill getting an A in the first course, and let S represent Averill getting an A in the second course.

$P(F) = \frac{3}{3+8}$ $\qquad$ $P(S) = \frac{6}{6+11}$

$P(F) = \frac{3}{11}$ $\qquad$ $P(S) = \frac{6}{17}$

$P(F) = 27.272...\%$ $\qquad$ $P(S) = 35.294...\%$

Averill should take the second course, because the probability of getting an A is higher.

8. Let C represent Cameron and Wyatt being chosen for president and secretary, and let O represent all possible choices. Because order is important, there are 2 ways that Cameron and Wyatt can be chosen for president and secretary. The total number of ways that 2 people can be chosen is ${}_9P_2$.

$n(O) = {}_9P_2$

$n(O) = \frac{9!}{(9-2)!}$

$n(O) = \frac{9!}{7!}$

$n(O) = \frac{9 \cdot 8 \cdot 7!}{7!}$

$n(O) = 9 \cdot 8$

$n(O) = 72$

Now determine the probability.

$P(C) = \frac{n(C)}{n(O)}$

$P(C) = \frac{2}{72}$

$P(C) = \frac{1}{36}$

The probability Cameron and Wyatt will be chosen for president and secretary is $\frac{1}{36}$, or about 0.0278 or 2.78%.

9. Let M represent a team containing Marina and MacKenzie, and let O represent all possible teams. The number of ways that Marina and MacKenzie can be placed on the team is ${}_4P_2$, and the number of ways to fill the other 2 spots on the team is ${}_9P_2$. Therefore, the total number of teams containing Marina and MacKenzie is ${}_4P_2 \cdot {}_9P_2$.

$n(M) = {}_4P_2 \cdot {}_9P_2$

$n(M) = \frac{4!}{(4-2)!} \cdot \frac{9!}{(9-2)!}$

$n(M) = \frac{4!}{2!} \cdot \frac{9!}{7!}$

$n(M) = \frac{4 \cdot 3 \cdot 2!}{2!} \cdot \frac{9 \cdot 8 \cdot 7!}{7!}$

$n(M) = 4 \cdot 3 \cdot 9 \cdot 8$

$n(M) = 864$

The total number of possible teams is ${}_{11}P_4$.

$n(O) = {}_{11}P_4$

$n(O) = \frac{11!}{(11-4)!}$

$n(O) = \frac{11!}{7!}$

$n(O) = \frac{11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7!}$

$n(O) = 11 \cdot 10 \cdot 9 \cdot 8$

$n(O) = 7920$

Now determine the probability.

$P(M) = \frac{n(M)}{n(O)}$

$P(M) = \frac{864}{7920}$

$P(M) = \frac{6}{55}$

The probability Marina and MacKenzie will be chosen to be on the relay team is $\frac{6}{55}$, or about 0.109 or 10.9%.

10. Let A represent a password containing the letters A, R, and T, and let O represent all possible passwords.
a) There are 3! ways of arranging the letters A, R, and T, and there are ${}_{10}P_4$ ways of arranging the four digits. Therefore, the number of passwords that contain the letters A, R, and T is $3! \cdot {}_{10}P_4$.

$n(A) = 3! \cdot {}_{10}P_4$

$n(A) = 3! \cdot \frac{10!}{(10-4)!}$

$n(A) = 3! \cdot \frac{10!}{6!}$

$n(A) = 3 \cdot 2 \cdot 1 \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!}$

$n(A) = 3 \cdot 2 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

The total number of passwords containing three capital letters followed by four digits, without repetition, is ${}_{26}P_3 \cdot {}_{10}P_4$.

$n(O) = {}_{26}P_3 \cdot {}_{10}P_4$

$n(O) = \frac{26!}{(26-3)!} \cdot \frac{10!}{(10-4)!}$

$n(O) = \frac{26!}{23!} \cdot \frac{10!}{6!}$

$n(O) = \frac{26 \cdot 25 \cdot 24 \cdot 23!}{23!} \cdot \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6!}{6!}$

$n(O) = 26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7$

Now determine the probability.

$P(A) = \frac{n(A)}{n(O)}$

$P(A) = \frac{3 \cdot 2 \cdot 10 \cdot 9 \cdot 8 \cdot 7}{26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8 \cdot 7}$

$P(A) = \frac{3 \cdot 2}{26 \cdot 25 \cdot 24}$

$P(A) = \frac{1}{26 \cdot 25 \cdot 4}$

$P(A) = \frac{1}{2600}$

The probability that a password chosen at random will contain the letters A, R, and T is $\frac{1}{2600}$, or about 0.000 385 or 0.0385%.

b) There are 3! ways of arranging the letters A, R, and T, and there are 10^4 ways of arranging the four digits. Therefore, the number of passwords that contain the letters A, R, and T is $3! \cdot 10^4$.

$n(A) = 3! \cdot 10^4$

$n(A) = 3 \cdot 2 \cdot 1 \cdot 10\,000$

$n(A) = 60\,000$

The total number of passwords containing three capital letters followed by four digits, allowing repetition, is $26^3 \cdot 10^4$

$n(O) = 26^3 \cdot 10^4$

$n(O) = 175\,760\,000$

Now determine the probability.

$P(A) = \frac{n(A)}{n(O)}$

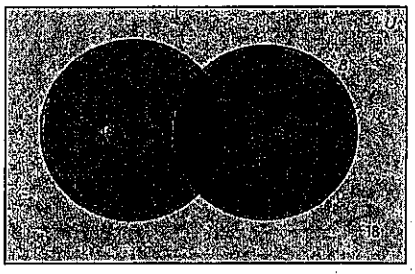
$P(A) = \frac{60000}{175760000}$

$P(A) = \frac{3}{8788}$

The probability that a password chosen at random will contain the letters A, R, and T is $\frac{3}{8788}$, or about 0.000 341 or 0.0341%.

11. a) These events are not mutually exclusive; e.g., There are prime numbers that are also odd numbers.
b) These events are mutually exclusive; e.g., You cannot roll a 6 and an 8 at the same time.
c) These events are mutually exclusive; e.g., You cannot eat a peach and an apple at the same time.

12. a) Let A represent a face card, and let B represent a spade.



b) These events are not mutually exclusive. This is because there are 6 cards that are both face cards and spades (2 jacks, 2 queens and 2 kings).

c) $P(A) = \frac{24}{48}$ $P(B) = \frac{12}{48}$

$P(A \cap B) = \frac{6}{48}$

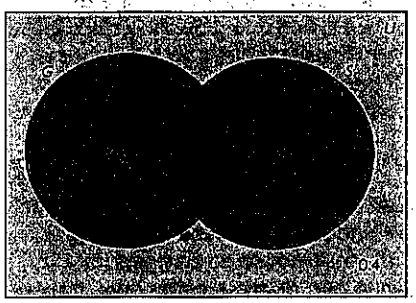
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = \frac{24}{48} + \frac{12}{48} - \frac{6}{48}$

$P(A \cup B) = \frac{30}{48}$

The probability that Hunter will draw a face card or a spade is $\frac{30}{48}$, 0.625 or 62.5%.

13. a) Let G represent exercising on Sunday, and let S represent shopping on Sunday.

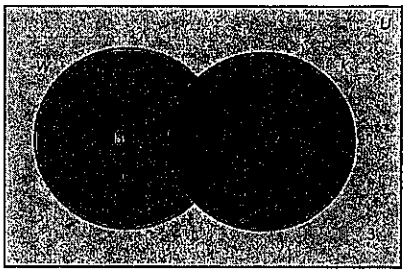


b) The two events are not mutually exclusive, because the probability Mya will exercise and shop on Sunday is not equal to 0 (it is equal to 0.2).

c)
$P(G) = 0.5$
$P(S) = 0.3$
$P(G \cap S) = 0.2$
$P(G \cup S) = P(G) + P(S) - P(G \cap S)$
$P(G \cup S) = 0.5 + 0.3 - 0.2$
$P(G \cup S) = 0.6$

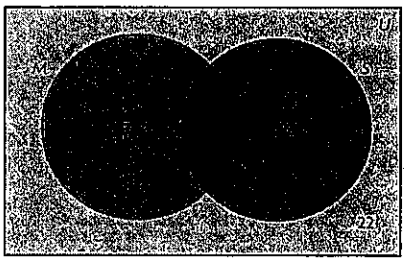
The probability Mya will do one of these activities on Sunday is 0.6, or 60%.

14. e.g., Suppose 6 students can both ski and swim. What is the probability that a randomly selected student cannot ski or swim. Let W = {students who swim} and K = {students who ski}.



Solution: The total number of students who can ski, swim or ski and swim is 13 + 6 + 2 = 21. There are 3 students who cannot ski or swim. The probability that a randomly selected student cannot ski or swim is $\frac{3}{24} = \frac{1}{8}$, 0.125 or 12.5%.

15. e.g., Out of 50 retail outlets, 19 are holding sales this month. 15 outlets sell only sustainably manufactured items, and 6 of these are holding sales this month. What is the probability that a retail outlet sells some non-sustainably manufactured items and is having a sale?
M = {outlets selling sustainably manufactured items}
S = {outlets having sales}



Solution: Let R represent retail outlets having sales that sell some non-sustainably manufactured items. From the Venn diagram we see 13 retail outlets that sell some non-sustainably manufactured items are having sales.

$P(R) = \frac{13}{50}$ The probability is $\frac{13}{50}$, 0.26 or 26%.

16. Let B represent a black sock, and let W represent a white sock. Let F represent Parker pulling a black sock then a white sock, and let G represent Parker pulling a white sock then a black sock. There are two ways to draw a white sock and a black sock.

$P(B) = \frac{8}{18}$ $P(F) = P(B) \cdot P(W|B)$

$P(B) = \frac{4}{9}$ $P(F) = \frac{4}{9} \cdot \frac{10}{17}$

$P(W|B) = \frac{10}{17}$ $P(F) = \frac{40}{153}$

$$P(W)=\frac{10}{18} \qquad P(G)=P(W)\cdot P(B|W)$$

$$P(W)=\frac{5}{9} \qquad P(G)=\frac{5}{9}\cdot\frac{8}{17}$$

$$P(B|W)=\frac{8}{17} \qquad P(G)=\frac{40}{153}$$

$$P(B\cap W)=P(F)+P(G)$$

$$P(B\cap W)=\frac{40}{153}+\frac{40}{153}$$

$$P(B\cap W)=\frac{80}{153}$$

The probability that Parker will pull out a pair of mismatched socks is $\frac{80}{153}$, or about 0.523 or 52.3%.

17. Let L represent a plane leaving from Winnipeg on time, and let A represent a plane arriving in Calgary on time.

$$P(L) = 0.70$$
$$P(L\cap A) = 0.56$$
$$P(A\mid L) = P(A|L)=\frac{P(L\cap A)}{P(L)}$$
$$P(A|L)=\frac{0.56}{0.70}$$
$$P(A|L)=0.8$$

The probability a plane will arrive in Calgary on time, given it left Winnipeg on time, is 0.8, or 80%.

18. Let R represent the first card drawn being red, and let B represent the second card drawn being black.

$$P(R)=\frac{20}{40} \qquad P(R\cap B)=P(R)\cdot P(B|R)$$

$$P(R)=\frac{1}{2} \qquad P(R\cap B)=\frac{1}{2}\cdot\frac{20}{39}$$

$$P(B|R)=\frac{20}{39} \qquad P(R\cap B)=\frac{20}{78}$$

The probability the first card is red and the second card is black is $\frac{20}{78}$, or about 0.256 or 25.6%.

19. Let S represent using a stair machine, and let B represent going to a body sculpt class.

$$P(S)=\frac{1}{3} \qquad P(B)=\frac{1}{2}$$

$$P(S\cap B)=P(S)\cdot P(B)$$

$$P(S\cap B)=\frac{1}{3}\cdot\frac{1}{2}$$

$$P(S\cap B)=\frac{1}{6}$$

The probability that Peyton will use a stair machine and take a body sculpting class when she next visits the gym is $\frac{1}{6}$, or about 0.167 or 16.7%.

20. If the events are independent, then the product of the individual probabilities will equal the probability of both the events occurring.

$$P(A\cap B) = P(A)\cdot P(B)$$
$$0.3 = 0.5\cdot 0.6$$
$$0.3 = 0.3$$

Yes, these events are independent.

21. Let F represent passing French, and let C represent passing chemistry.

a)
$$P(F) = 0.7$$
$$P(C) = 0.6$$
$$P(F\cap C) = P(F)\cdot P(C)$$
$$P(F\cap C) = 0.7\cdot 0.6$$
$$P(F\cap C) = 0.42$$

The probability Tanya will pass both French and chemistry is 0.42, or 42%

b)
$$P(C') = 0.4$$
$$P(F\cap C') = P(F)\cdot P(C')$$
$$P(F\cap C') = 0.7\cdot 0.4$$
$$P(F\cap C') = 0.28$$

The probability Tanya will pass French but fail chemistry is 0.28, or 28%.

c)
$$P(F') = 0.3$$
$$P(F'\cap C') = P(F')\cdot P(C')$$
$$P(F'\cap C') = 0.3\cdot 0.4$$
$$P(F'\cap C') = 0.12$$

The probability Tanya will fail both French and chemistry is 0.12, or 12%.

Chapter Task, page 207

A. I chose Gin Rummy.

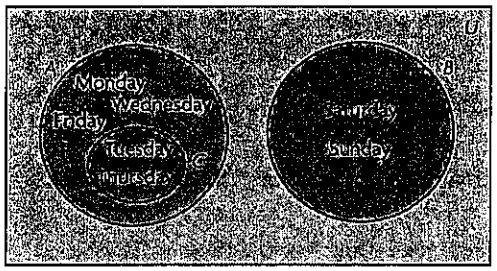
B. For one round, I have A♥, 2♥, 3♥, 6♥, 6♦, 8♣, 9♣, 10♦, J♣, and K♥ in my hand. I know that my opponent has picked up the K♣ and the 7♠. I also know that he has discarded the 9♦ and the J♥. The J♥ is currently face up. There are still 20 unturned cards in the deck.

C. There are, at most, seven hearts left in the deck, including the unknown cards in my opponent's hand. I could pick up the J♥, hoping to get either another jack or the Q♥ during a later play; or I could draw a face-down card. The conditional probability of drawing the 4♥ or 5♥ is, at most, $\frac{2}{20}$ or $\frac{1}{10}$. This would strengthen my hearts, potentially getting a longer run; but if I drew the 5♥ and did not get the 4♥ during a later turn, my hand would not be strengthened. The probability of drawing one of these cards now and one on the next turn would be, at most, $\left(\frac{1}{10}\right)\cdot\left(\frac{1}{19}\right)$ or $\frac{1}{190}$ (I used 19 because I assumed that my opponent would pick up my discarded card. I would use 18 if he drew a card from the deck.) If I pick up the J♥, then there are possibly three cards that would strengthen my hand remaining in the deck: the Q♥, the J♣, and the J♦. The probability that I will get one of those

cards in my next draw is, at most, $\frac{3}{19}$. It makes sense to pick up the J♥. I still need to discard one of the cards in my hand. Since my opponent threw away the 9♦, I am going to guess that he may not have cards near the 9♦ to make a run, so it makes sense to discard the 10♦.
D. One strategy is to observe which card your opponent collects and to think about why he or she wants this card. Does your opponent want it to make a set or a run? If so, you might want to prevent your opponent from getting cards that would allow him or her to complete the set. Another strategy is to collect a card you do not really need to fool your opponent into thinking that you are looking for this type of card.

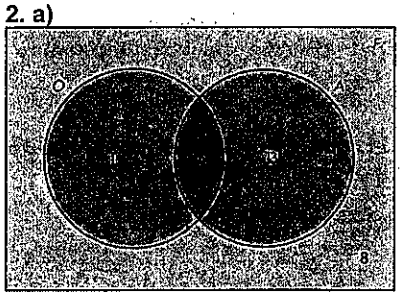
1–3 Cumulative Review, page 210

1. a)



b) A and B have no common elements, so they are disjoint sets.
c) i) false; e.g., A and B are disjoint sets, so A is not a subset of B.
ii) true; C is entirely inside A, so it a subset of A.
iii) true; B contains all the elements that are not in A, so B is the inverse of A.
iv) false;
$n(A) + n(B) + n(C) = 5 + 2 + 2$
$n(A) + n(B) + n(C) = 9$
but $n(U) = 7 \neq 9$.
This is because A includes C, so $n(C)$ is counted twice.

2. a)



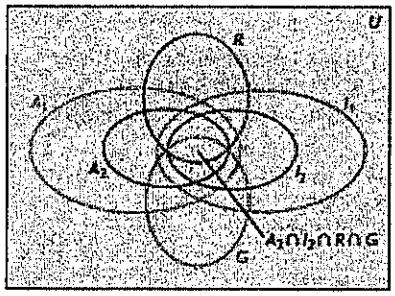
b) There are 35 students in all, and 8 like neither fruit, so 35 – 8 = 27 students like oranges or apples.
c) 27 – 14 = 13, so 13 students like only apples.
27 – 26 = 1, so 1 student likes only oranges.
27 – 13 – 1 = 13, so 13 students like oranges and apples. 14 students like only apples or only oranges.

3. a) e.g., odd whole numbers less than 100 and even whole numbers less than 100
b) e.g., odd whole numbers less than 100 and prime numbers less than 100

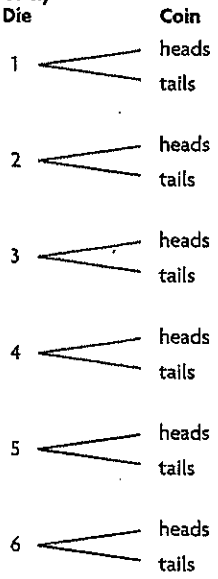
4. Let S = {campers who learned singing},
D = {campers who learned dancing},
A = {students who learned acting}
Then: $n(S) = 35$, $n(D) = 38$, $n(A) = 33$
$n(S \cap D) = 21$, $n(D \cap A) = 23$, $n(S \cap A)$
$n(D \cup S \cup A) = 56$

By the Principle of Exclusion and Inclusion:
$n(D \cup S \cup A)$
$= n(S) + n(D) + n(A) - n(S \cap D) - n(D \cap A) - n(S \cap A)$
$\quad + n(D \cap S \cap A)$
$56 = 35 + 38 + 33 - 21 - 23 - 18 + n(D \cap S \cap A)$
$12 = n(D \cap S \cap A)$
So, 12 campers learned singing, dancing, and acting.

5. e.g., Some search terms Guido might use are ancient ruins, southern Italy, Roman, and Greek.
Let A_1 = {ancient ruins}
A_2 = {"ancient ruins"}
I_1 = {southern Italy}
I_2 = {"southern Italy"}
R = {Roman}
G = {Greek}



6. a)



b) Rolling a die has 6 outcomes: 1, 2, 3, 4, 5, 6; tossing a coin has 2 outcomes: heads, tails.
By the Fundamental Counting Principle, there are $6 \cdot 2 = 12$ combined outcomes, which is how many there are in the tree diagram.

7. a) There are 15! ways, or 1 307 674 368 000 ways.
b) Out of 15 members, 3 can be chosen, with regard for order, in ${}_{15}P_3 = 2730$ ways.
c) Out of 15 members, Jill, and 2 other members can be chosen in ${}_{14}C_2 = 91$ ways.

8. There were 13 + 5 = 18 games played, so there are ${}_{18}C_{13} = 8568$ ways in which 13 wins could have occurred.

9.
$$
\begin{aligned}
{}_nC_4 &= {}_nP_3 \\
\frac{n!}{(n-4)!4!} &= \frac{n!}{(n-3)!} \\
\frac{n(n-1)(n-2)(n-3)(n-4)!}{(n-4)!4!} &= \frac{n(n-1)(n-2)(n-3)!}{(n-3)!} \\
\frac{n(n-1)(n-2)(n-3)}{4!} &= n(n-1)(n-2) \\
n(n-1)(n-2)(n-3) &= 24n(n-1)(n-2) \\
n-3 &= 24 \\
n &= 27
\end{aligned}
$$

10. a) There are 7 + 9 = 16 people in total.
If 3 are parents and 3 are students, then:
${}_7C_3 \cdot {}_9C_3 = 35 \cdot 84$ or 2940 ways in this can happen.

b) If there is at least 1 student, then there can be:
• 5 parents and 1 student:
${}_7C_5 \cdot {}_9C_1 = 21 \cdot 9$ or 189 ways

• 4 parents and 2 students:
${}_7C_4 \cdot {}_9C_2 = 35 \cdot 36$ or 1260 ways

• 3 parents and 3 students:
${}_7C_3 \cdot {}_9C_3 = 35 \cdot 84$ or 2940 ways

• 2 parents and 4 students:
${}_7C_2 \cdot {}_9C_4 = 21 \cdot 126$ or 2646 ways

• 1 parent and 5 students:
${}_7C_1 \cdot {}_9C_5 = 7 \cdot 126$ or 882 ways

• 0 parents and 6 students:
${}_9C_6 = 84$ ways

Total number of ways
= 189 + 1260 + 2940 + 2646 + 882 + 84
= 8001
There are 8001 ways in which the committee can have at least 1 student.

11. Since each student is equally likely to win, then Sonya is just as likely to finish in the top three (the top half) as in the bottom three (the bottom half). The probability Sonya will finish in the top three is 0.5.

12. a) There are ${}_{52}C_{13} = 635\,013\,559\,600$ ways in which 13 cards can be selected from a deck of 52 cards. There is only one way in which all of these 13 cards can be hearts. The probability that a player is dealt all hearts is $\frac{1}{635\,013\,559\,600}$.

b)
$$
\begin{aligned}
\frac{\begin{pmatrix}\text{all kings} \\ \text{other players's cards}\end{pmatrix}}{\text{all possible hands}} &= \frac{\binom{48}{39}}{\binom{52}{13}} \\
&= \frac{1\,677\,106\,640}{635\,013\,559\,600}
\end{aligned}
$$
The probability is $\frac{1\,677\,106\,640}{635\,013\,559\,600}$.

13. I made a table of the sums. There are 36 possible results.

Die 1 / Die 2	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

a) The sum is 3 or 12 on three occasions.
The probability of this is $\frac{3}{36}$ or about 0.0833.

b) The sum is odd or the sum is greater than 7 on 27 occasions. The probability of this is $\frac{27}{36}$ or 0.75.

c) There are 6 occasions on which the first die is 1. Of these, there are three occasions on which the sum is odd. The probability of this is $\frac{3}{6}$ or 0.5.

d) There are 2 occasions out of 36 when the first die is less than 3 and the second is greater than 5. The probability of this is $\frac{2}{36}$ or about 0.0556.

14. a) Jan can win at least one game in three ways:
• She wins both games:

$P(\text{win both games})$

$= P(\text{win game 1}) \cdot P(\text{win game 2} \mid \text{won game 1})$

$= \frac{1}{2} \cdot \left(\frac{1}{2} + \frac{1}{8}\right)$

$= \left(\frac{1}{2}\right) \cdot \left(\frac{5}{8}\right)$

$= \frac{5}{16}$ or 0.3125

• She wins game 1 and loses game 2:

$P(\text{win game 1, lose game 2})$

$= P(\text{win game 1}) \cdot P(\text{lose game 2} \mid \text{won game 1})$

$= \frac{1}{2} \cdot \left(1 - \frac{5}{8}\right)$

$= \left(\frac{1}{2}\right) \cdot \left(\frac{3}{8}\right)$

$= \frac{3}{16}$ or 0.1875

• She lose game 1 and wins game 2:

$P(\text{lose game 1, win game 2})$

$= P(\text{lose game 1}) \cdot P(\text{win game 2} \mid \text{lost game 1})$

$= \frac{1}{2} \cdot \left(\frac{1}{2} - \frac{1}{4}\right)$

$= \left(\frac{1}{2}\right) \cdot \left(\frac{1}{4}\right)$

$= \frac{1}{8}$ or 0.125

The total of these probabilities is
0.3125 + 0.1875 + 0.125 = 0.625
The probability Jan will win at least one game is 0.625.

b) $P(\text{Stan wins both games})$

$= 1 - P(\text{Jan wins at least one game})$

$= 1 - 0.625$

$= 0.375$

15. a) There are 24 cards, of which 4 are queens. The probability of drawing a queen on the first draw is $\frac{4}{24} = \frac{1}{6}$. If the card is replaced, the probability of drawing a queen on the second draw is the same, $\frac{1}{6}$.

So, $P(2 \text{ queens}) = \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{36}$ or about 0.0278.

b) The probability of drawing a queen on the first draw remains the same, $\frac{1}{6}$. If this card is not replaced, then there is one less queen to draw and one less card to draw from. The probability of drawing a queen on the second draw is $\frac{4-1}{24-1} = \frac{3}{23}$. So,

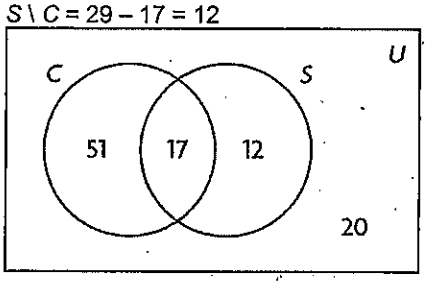
$P(2 \text{ queens}) = \frac{4}{24} \cdot \frac{3}{23} = \frac{12}{552}$ or about 0.0217.

Chapter 3 Diagnostic Test, TR page 195

1. Let U be the universal set.

$C \setminus S = 68 - 17 = 51$

$S \setminus C = 29 - 17 = 12$



2. a) The number of ways to roll a 5 is 1; the number of outcomes for the die is 6. The number of ways to toss a tail is 1; the number of outcomes is 2.

$\left(\frac{1}{6}\right) \cdot \left(\frac{1}{2}\right) = \frac{1}{12}$

b) The number of ways to roll an even number is 3; the number of outcomes is 6. The number of ways to toss a head is 1; the number of outcomes is 2.

$\left(\frac{3}{6}\right) \cdot \left(\frac{1}{2}\right) = \left(\frac{1}{2}\right) \cdot \left(\frac{1}{2}\right)$ or $\frac{1}{4}$

3. The total volume of the cleaning compound is volume of concentrate and volume of water: 2 + 5. The ratio of volume of concentrate to total volume of the cleaning compound is 2 : 7.

4. There are ten letters in STATISTICS.
S is repeated three times, T is repeated three times, and I is repeated twice. The letters can be arranged 10! ways. To eliminate arrangements that would be the same because of the repeating letters, divide by 3!, 3! and 2!.

$\frac{10!}{3! \cdot 3! \cdot 2!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3! \cdot 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1}$

$\frac{10!}{3! \cdot 3! \cdot 2!} = \frac{10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4}{3 \cdot 2 \cdot 2}$

$\frac{10!}{3! \cdot 3! \cdot 2!} = \frac{604\,800}{12}$

$\frac{10!}{3! \cdot 3! \cdot 2!} = 50\,400$

The letters can be arranged in 50 400 ways.

5. $_9P_5 = \frac{9!}{(9-5)!}$

$_9P_5 = \frac{9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4!}$

$_9P_5 = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5$

$_9P_5 = 15\,120$

Remi can design the pendant in 15 120 ways.

6. $_{24}C_5 = \frac{24!}{(24-5)! \cdot 5!}$

$_{24}C_5 = \frac{24 \cdot 23 \cdot 22 \cdot 21 \cdot 20 \cdot 19!}{19! \cdot 5 \cdot 4 \cdot 3 \cdot 2}$

$_{24}C_5 = \frac{24 \cdot 23 \cdot 22 \cdot 21 \cdot 20}{5 \cdot 4 \cdot 3 \cdot 2}$

$_{24}C_5 = 42\,504$

The students can be selected in 42 504 ways.

7. Agree. e.g., The outcome of tossing the coin has no bearing on the outcome of tossing the die. This means these are independent events.

Review of Terms and Connections, TR page 197

1. a) ix) If one thing can be done in five ways and another thing can be done in three ways, then both things can be done in 5 · 3 or 15 ways, according to the **Fundamental Counting Principle.**

b) ii) Two sets that have no elements in common are **mutually exclusive.**

c) v) The value $\frac{3}{10}$, written as a **ratio,** is 3 : 10.

d) iii) An arrangement of objects in a definite order is a **permutation** of the objects.

e) vii) If the product 5 · 4 · 3 · 2 · 1 is written as 5!, it is written in **factorial notation.**

f) iv) A group of objects in which order does not matter is a **combination** of the objects.

g) i) The set of all possible outcomes is called the **sample space.**

h) vi) Brenda tossed a coin 10 times. It turned up tails 8 times. She used these results to determine that the probability of tossing tails is $\frac{8}{10}$. This is an example of **experimental** probability.

i) viii) Serge says that the probability of tossing heads with a fair coin is $\frac{1}{2}$, because the likelihood that the coin will land heads is equal to the likelihood that it will land tails. This is an example of **theoretical** probability.

2. a) $_8P_5 = \frac{8!}{(8-5)!}$

$_8P_5 = \frac{8!}{3!}$

$_8P_5 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3!}{3!}$

$_8P_5 = 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4$

$_8P_5 = 6720$

b) $_{18}P_2 = \frac{18!}{(18-2)!}$

$_{18}P_2 = \frac{18!}{16!}$

$_{18}P_2 = \frac{18 \cdot 17 \cdot 16!}{16!}$

$_{18}P_2 = 18 \cdot 17$

$_{18}P_2 = 306$

c) $_4P_4 = \frac{4!}{(4-4)!}$

$_4P_4 = \frac{4!}{0!}$

$_4P_4 = \frac{4 \cdot 3 \cdot 2 \cdot 1}{1}$

$_4P_4 = 4 \cdot 3 \cdot 2$

$_4P_4 = 24$

d) $_{11}C_8 = \frac{11!}{(11-8)!\,8!}$

$_{11}C_8 = \frac{11!}{3! \cdot 8!}$

$_{11}C_8 = \frac{11 \cdot 10 \cdot 9 \cdot 8!}{3 \cdot 2 \cdot 1 \cdot 8!}$

$_{11}C_8 = \frac{11 \cdot 10 \cdot 9}{3 \cdot 2}$

$_{11}C_8 = \frac{990}{6}$

$_{11}C_8 = 165$

e) $_{11}C_3 = {}_{11}C_8$, since 3 + 8 = 11

$_{11}C_3 = 165$

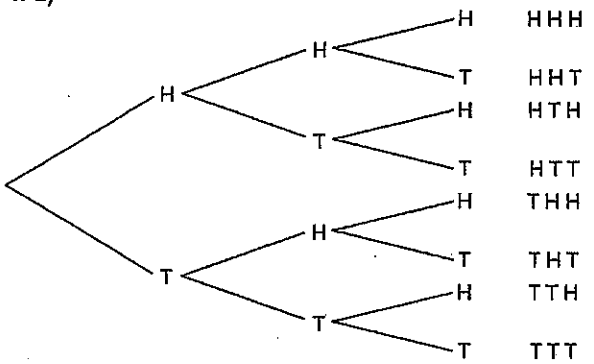
f) $_{24}C_{24} = 1$, since there is only one way to choose 24 from 24.

3. a)

Products of the Rolls of Two Dice						
Die 1/ Die 2	**1**	**2**	**3**	**4**	**5**	**6**
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36

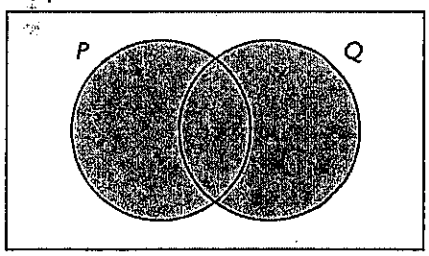
b) There are four ways to roll 12 and 36 possible outcomes. The probability of rolling a product of 12 is $\frac{4}{36}$ or $\frac{1}{9}$.

4. a)

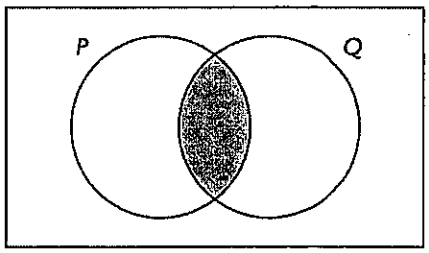


b) There are three ways to toss one head and two tails, and eight possible outcomes: $P(1\text{ H}, 2\text{T}) = \frac{3}{8}$.

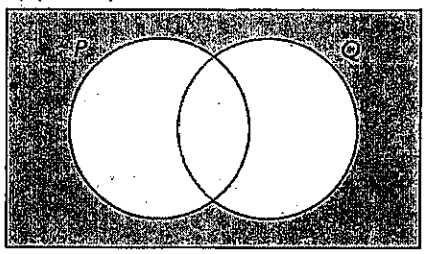
5. a) $P \cup Q$



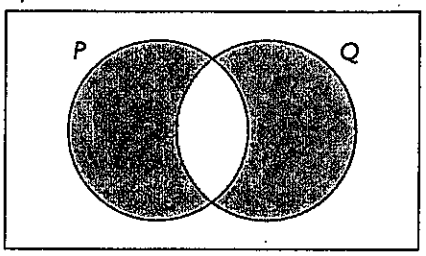
b) $P \cap Q$



c) $(P \cup Q)'$



d) $P \setminus Q \cup Q \setminus P$



6. a) e.g.,

	Train A					
Train B	**7:11**	**7:12**	**7:13**	**7:14**	**7:15**	**7:16**
7:13	11/13	12/13	13/13	14/13	15/13	16/13
7:14	11/14	12/14	13/14	14/14	15/14	16/14
7:15	11/15	12/15	13/15	14/15	15/15	16/15
7:16	11/16	12/16	13/16	14/16	15/16	16/16
7:17	11/17	12/17	13/17	14/17	15/17	16/17

b) i) There are four ways the trains will arrive in the same minute, and there are 30 possible outcomes: the probability the trains will arrive in the same minute is $\frac{4}{30}$.

ii) There are 20 ways train B will arrive at least one minute after train A has arrived, and there are 30 possible outcomes: the probability train B will arrive at least one minute after train A is $\frac{20}{30}$.

7. Since order does not matter, this is a combination problem.

$$_{12}C_5 = \frac{12!}{(12-5)!5!}$$

$$_{12}C_5 = \frac{12!}{7! \cdot 5!}$$

$$_{12}C_5 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{7! \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$$_{12}C_5 = \frac{12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{5 \cdot 4 \cdot 3 \cdot 2}$$

$$_{12}C_5 = \frac{95\,040}{120}$$

$$_{12}C_5 = 792$$

There are 792 ways Jasmine can invite 5 of her 12 friends to drive to the ski resort.

8. The first digit must be a vowel, so there are 5 options, the last must also be a vowel, without repeating the first so there are 4 options.
5 _, _, _, 4
For the middle three digits, there are 24 options left for the second, 23 for the third, and 22 for the fourth.
5, 24, 23, 22, 4
Using the Fundamental Counting Principle, there are $5 \cdot 24 \cdot 23 \cdot 22 \cdot 4$ or 242 880 five-character passwords that can be formed under these conditions.

9. Since order does not matter, this is a combination problem.

$${}_8C_4 = \frac{8!}{(8-4)!4!}$$

$${}_8C_4 = \frac{8!}{4! \cdot 4!}$$

$${}_8C_4 = \frac{8 \cdot 7 \cdot 6 \cdot 5 \cdot 4!}{4! \cdot 4 \cdot 3 \cdot 2 \cdot 1}$$

$${}_8C_4 = \frac{8 \cdot 7 \cdot 6 \cdot 5}{4 \cdot 3 \cdot 2}$$

$${}_8C_4 = \frac{1680}{24}$$

$${}_8C_4 = 70$$

There are 70 ways eight members can be chosen to chair four different committees.

10. There is one way to toss a 3, and six possible outcomes. The probability of tossing a 3 once is $\frac{1}{6}$.
Using the Fundamental Counting Principle:

$$\left(\frac{1}{6}\right)\left(\frac{1}{6}\right) = \frac{1}{36}$$

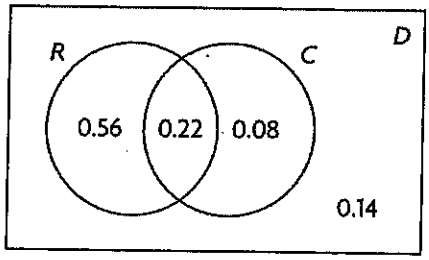
The probability of tossing a 3 both times when a 6-sided die is tossed twice is $\frac{1}{36}$.

Chapter 3 Test, TR page 202

1. The total number of outcomes can be determined from the odds: 9 + 6 = 15. From the odds against, 9 : 6, we know the number of ways the event can happen is 6. Therefore, the probability is $\frac{6}{15}$ or 0.4.

2. Germaine's probability of winning is $\frac{8}{8+5} = \frac{8}{13}$ or about 0.62. Gabriel's probability of winning is $\frac{7}{7+4} = \frac{7}{11}$ or about 0.64. Gabriel should start, because his record is better.

3. a) Let R represent serving rice. Let C represent serving carrots.
$P(R) + P(C) + P(R \cup C)' = 0.78 + 0.30 + 0.14 = 1.22$
The total cannot exceed 1, so $P(R \cap C)$ must be 0.22.
$P(R \setminus C) = 0.78 - 0.22 = 0.56$
$P(R \setminus C) = 0.30 - 0.22 = 0.08$



b) $P(R \cap C) = 0.22$

4. The number of ways of selecting 7 spades from a deck of cards is ${}_{13}C_7$. The number of ways of selecting any 7 cards from a deck of cards is ${}_{52}C_7$.

$${}_{13}C_7 = \frac{13!}{(13-7)!7!}$$

$${}_{13}C_7 = \frac{13!}{6! \cdot 7!}$$

$${}_{13}C_7 = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8 \cdot 7!}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 7!}$$

$${}_{13}C_7 = \frac{13 \cdot 12 \cdot 11 \cdot 10 \cdot 9 \cdot 8}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}$$

$${}_{13}C_7 = \frac{1\,235\,520}{720}$$

$${}_{13}C_7 = 1716$$

$${}_{52}C_7 = \frac{52!}{7!(52-7)!}$$

$${}_{52}C_7 = \frac{52!}{7! \cdot 45!}$$

$${}_{52}C_7 = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46 \cdot 45!}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 \cdot 45!}$$

$${}_{52}C_7 = \frac{52 \cdot 51 \cdot 50 \cdot 49 \cdot 48 \cdot 47 \cdot 46}{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2}$$

$${}_{52}C_7 = 133\,784\,560$$

$$P(7 \text{ spades}) = \frac{{}_{13}C_7}{{}_{52}C_7}$$

$$P(7 \text{ spades}) = \frac{1716}{133\,784\,560}$$

The probability of selecting 7 spades is $\frac{1716}{133\,784\,560}$ or about 0.000 012 8.

5. Assume that the 3 books are arranged as a group. There are 7 other books, so I can arrange 8 objects as ${}_8P_8$ or 8!. I can also arrange the 3 books within their group as 3!.
The total number of ways to arrange all 10 books is 10!.

$$P(\text{the 3 books are together}) = \frac{8! \cdot 3!}{10!}$$

$$P(\text{the 3 books are together}) = \frac{8! \cdot 3 \cdot 2 \cdot 1}{10 \cdot 9 \cdot 8!}$$

$$P(\text{the 3 books are together}) = \frac{3 \cdot 2}{10 \cdot 9}$$

$$P(\text{the 3 books are together}) = \frac{6}{90}$$

$$P(\text{the 3 books are together}) = \frac{1}{15}$$

The probability that the three books will be placed together is $\frac{1}{15}$ or about 0.07.

6. a) This is a conditional probability problem.

$$P(2 \text{ red}) = P(1\text{st red}) \cdot P(2\text{nd red} \mid 1\text{st red})$$

$$P(2 \text{ red}) = \left(\frac{7}{12}\right)\left(\frac{6}{11}\right)$$

$$P(2 \text{ red}) = \frac{42}{132}$$

$$P(2 \text{ red}) = 0.318...$$

The probability both marbles are red is about 0.32.

b) $P(2 \text{ blue}) = P(1\text{st blue}) \cdot P(2\text{nd blue} \mid 1\text{st blue})$

$$P(2 \text{ blue}) = \left(\frac{5}{12}\right)\left(\frac{4}{11}\right)$$

$$P(2 \text{ blue}) = \frac{20}{132}$$

$$P(2 \text{ blue}) = 0.151...$$

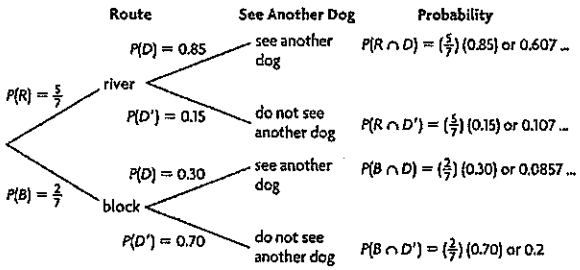
The probability both marbles are blue is about 0.15.

c) $P(1 \text{ red, } 1 \text{ blue}) = 1 - P(2 \text{ blue}) - P(2 \text{ red})$
$P(1 \text{ red, } 1 \text{ blue}) = 1 - 0.318... - 0.151...$
$P(1 \text{ red, } 1 \text{ blue}) = 0.530...$
The probability of drawing one red and one blue marble is about 0.53.

7. a) Let R represent walking by the river, $P(R) = \frac{5}{7}$.

Let B represent walking around the block, $P(B) = \frac{2}{7}$.

Let D represent seeing another dog.
$P(D \mid R) = 0.85$
$P(D \mid B) = 0.30$



b) $P(\text{see another dog}) = P(R \cap D) + P(B \cap D)$

$P(\text{see another dog}) = P(R) \cdot P(D \mid R) + P(B) \cdot P(D \mid B)$

$$P(\text{see another dog}) = \left(\frac{5}{7}\right) \cdot (0.85) + \left(\frac{2}{7}\right) \cdot (0.30)$$

$P(\text{see another dog}) = 0.607... + 0.085...$

$P(\text{see another dog}) = 0.692...$

The probability that Logan and his dog will see another dog is about 0.69.